

## Formulas

1. Discrete convolution  $x * y_k = \sum_{m=-\infty}^{\infty} x_{k-m} y_m = \sum_{m=-\infty}^{\infty} x_m y_{k-m}$
2. High pass decomposition:  $h_k = \frac{1}{2}(-1)^k p_{k+1}$
3. Low pass decomposition:  $\ell_k = \frac{1}{2}p_{-k}$
4. High pass reconstruction:  $\tilde{h}_k = q_k, q_k = (-1)^k p_{1-k}$
5. Low pass reconstruction:  $\tilde{\ell}_k = p_k$
6. Scaling function:  $\phi(x) = \sum_{k=-\infty}^{\infty} p_k \phi(2x-k), p_k = 2 \int_{-\infty}^{\infty} \phi(2x-k) \phi(x) dx$
7. Wavelet:  $\psi(x) = \sum_{k=-\infty}^{\infty} q_k \phi(2x-k), q_k = (-1)^k p_{1-k}$
8. Decomposition formulas:  $a_\ell^{j-1} = \frac{1}{2} \sum_{k=-\infty}^{\infty} p_{k-2\ell} a_k^j, b_\ell^{j-1} = \frac{1}{2} \sum_{k=-\infty}^{\infty} q_{k-2\ell} a_k^j, q_k = (-1)^k p_{1-k}$
9. Reconstruction formula:  $a_k^j = \sum_{\ell=-\infty}^{\infty} p_{k-2\ell} a_\ell^{j-1} + \sum_{\ell=-\infty}^{\infty} q_{k-2\ell} b_\ell^{j-1}, q_k = (-1)^k p_{1-k}$

## Fourier Transform Properties

1.  $\widehat{f}(\lambda) = \mathcal{F}[f](\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ix\lambda} dx.$
2.  $\frac{f(x^+) + f(x^-)}{2} = \mathcal{F}^{-1}[\widehat{f}](x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(\lambda) e^{ix\lambda} d\lambda.$
3.  $\mathcal{F}[x^n f(x)](\lambda) = i^n \widehat{f}^{(n)}(\lambda).$
4.  $\mathcal{F}[f^{(n)}](\lambda) = (i\lambda)^n \widehat{f}(\lambda).$
5.  $\mathcal{F}[f(bx - a)](\lambda) = \frac{1}{b} e^{-i\lambda a/b} \widehat{f}(\lambda/b).$
6.  $\mathcal{F}[\text{sinc}(x)](\lambda) = \begin{cases} \frac{1}{\sqrt{2\pi}}, & -\pi \leq \lambda \leq \pi \\ 0, & \text{otherwise} \end{cases}$