# Response to George's note

Eduardo Martín-Martínez[1,2] and José de Ramón[1]

[1] *Department of Applied Mathematics, University of Waterloo*

[2] *Perimeter Institute for Theoretical Physics, Waterloo*

## Abstract

What we do in this note with some level of detail can be summarized as follows:

- We do like the thinking in George's note, however we do have some concerns about it along the lines of what Bill already raised.
- We provide some detailed reasoning as to what is our concern, and in particular how we understand the so-called 'structureless' limit.
- As we go along George's points, we wrote some preliminaries and review to set notation and help translate the arguments in terms of Rindler excitations to UDW response in the long-time adiabatic limit. Also hopefully helpful to introduce some of the notions we use in our argument.
- The argument per-se starts in the point (26-end) in page 4.
- Summarizing: we believe that $\Omega \to 0$ is not a structureless limit in general, although there are some cases where it can be thought of as such.
- We give an example to visualize why this is the case and how (as pointed out by Bill) one arrives to the the unsatisfactory result that eventually the emission will be independent of the state of the field in that limit.
- We discuss in what way we believe that the UDW detector approach to this problem can maybe still be interesting.

In this note, we have structured our arguments going one by one over the points raised in George's note to try to make it easier to read. The numbers in parenthesis at the beginning of each paragraph correspond to the numbered points in George's original note. Apologies in advance for any possible misrepresentations or misunderstandings.

(1-6) We can accept the claim that the acceleration radiation may be associated with the zero frequency Rindler modes. However, we are suspicious of the relation between this and the Unruh effect, since a point-like charge perturbs the quantum field in a way that does not depend on the state of the field. This is along the lines of the same Argument Bill mentioned in his email.

A line of argument that seems appealing to us is a bit of a twist on the original argument: It may be argued that in states that do not exhibit the Unruh effect, (e.g. the Rindler vacuum), the RSET is divergent [1]. In this case we can imagine an argument of the kind: There may be no reasonable vacua (with reasonable expectation value of renormalized observables) unless the vacuum we consider is a state that displays Unruh effect (in the sense of being KMS with respect to the generator of boosts).

(7-10) All good. Just as a comment, there is a very nice paper that explores the different scales that enter in the switching function and how they affect the thermalization of particle detectors [2].

(11) Nothing to add.

(12-25) We agree with the calculation. Basically there is a splitting in the rotating and counter-rotating contributions with respect to the proper time, which are then associated with emission and absorption of Rindler quanta. However, we think that this association is only meaningful when the interaction is long and adiabatic, which is the regime in which 'excitation number non-preserving' contributions vanish. Indeed this is what it is outlined in the items (21-25).

In terms that are more familiar to us (less focused on Rindler quanta and more in the response of the detector itself), let us revisit the (Takagi style) response of a detector to a stationary state of the field. Consider a two-level UDW detector and the eigenbasis of its free Hamiltonian notated as $H_{\text{free}}\left|g\right\rangle = 0, H_{\text{free}}\left|e\right\rangle = \Omega\left|e\right\rangle$. An initial state for detector that is diagonal in this basis can be always written as

$$\hat{\rho}_{\text{D}}(\beta) = \frac{e^{-\beta \hat{H}_{\text{free}}}}{Z} = \frac{e^{-\beta\Omega}\left|e\right\rangle\!\left\langle e\right| + \left|g\right\rangle\!\left\langle g\right|}{1+e^{-\beta\Omega}} \tag{1}$$

with $\beta \in (-\infty, \infty)$. Consider an arbitrary field state $\hat{\rho}_\phi$. Consider the Unruh-DeWitt interaction Hamiltonian

$$\hat{H}_I = \lambda c(\tau/T)\hat{\mu}(\tau)\hat{\phi}(\mathsf{x}(\tau)), \tag{2}$$

where $c(\tau/T)$ is the switching function, and $T$ is the interaction duration timescale (that adimensionalizes $\tau$ in the switching function). The time evolution of the detector under the interaction Hamiltonian (2) is given by

$$\hat{\rho}'_{\text{D}} = \text{tr}_\phi\left(\hat{U}(\hat{\rho}_{\text{D}}\otimes\hat{\rho}_\phi)\hat{U}^\dagger\right) = \hat{\rho}_{\text{D}} + \lambda^2 T\frac{\mathcal{F}(\Omega) - e^{-\beta\Omega}\mathcal{F}(-\Omega)}{1+e^{-\beta\Omega}}(|e\rangle\langle e| - |g\rangle\langle g|) + \mathcal{O}(\lambda^4), \tag{3}$$

and $\lambda$ is the coupling constant. Further, we have defined the response function as

$$\mathcal{F}(\Omega) \coloneqq \frac{1}{T}\iint \text{d}\tau\text{d}\tau' c(\tau/T)c(\tau'/T)\left\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))\right\rangle e^{-\text{i}\Omega(\tau-\tau')}. \tag{4}$$

If the pull-back of the Wightman function is stationary in the proper time of the detector, i.e.

$$\left\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(\tau'))\right\rangle = \left\langle\hat{\phi}(\mathsf{x}(\tau-\tau'))\hat{\phi}(\mathsf{x}(0))\right\rangle, \tag{5}$$

then the the pull-back acts as a one dimensional distribution over the convolution of the switching functions, i.e.

$$\mathcal{F}(\Omega) = \frac{1}{T}\int \text{d}\tau \left\langle\hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(0))\right\rangle e^{-\text{i}\Omega\tau}\Big([\bar{c}*c](\tau/T)\Big). \tag{6}$$

where

$$[\bar{c}*c](\tau/T) \coloneqq \int \text{d}\tau' c((\tau'-\tau)/T)c(\tau'/T). \tag{7}$$

Therefore, the response function can be written in frequency representation as [2]

$$\mathcal{F}(\Omega) = T\int \text{d}\omega\tilde{\mathcal{W}}\left(\Omega+\omega\right)|\tilde{c}|^2(T\omega) \tag{8}$$

where

$$\tilde{\mathcal{W}}(\omega) = \int \mathrm{d}\tau \, \langle \hat{\phi}(\mathsf{x}(\tau))\hat{\phi}(\mathsf{x}(0))\rangle \, e^{-\mathrm{i}\omega\tau}. \tag{9}$$

Now, provided that the switching is nice enough, meaning that it decays in the frequency space faster than any polynomial up to some degree,

$$\lim_{T\to\infty} \mathcal{F}(\Omega) = \tilde{\mathcal{W}}(\Omega)\|c\|_2. \tag{10}$$

where $\|c\|_2$ is the $L^2$-norm of the switching. It is well known that for the Minkowski vacuum in 4 dimensions and accelerated trajectories [3]

$$\tilde{\mathcal{W}}(\Omega) = \frac{1}{2\pi}\frac{\Omega}{e^{\frac{2\pi\Omega}{a}} - 1} \tag{11}$$

In this sense, we understand the claim that the detector interacts with the Rindler modes, since the Fourier transform in (9) shows that the swithcing function is 'windowing' the (Rindler) frequency domain conjugate to the proper time $\tau$, and in the very long time limit, it selects one single 'Rindler' frequency.

(26-end) Our disagreement comes primarily form these items. We understand that the limit $\Omega/a \to 0$ not as a structureless limit, but as a high temperature/acceleration limit. Indeed, in this limit :

$$\tilde{\mathcal{W}}(\Omega) \sim \frac{a}{4\pi^2} \tag{12}$$

and, besides the long-time adiabatic limit

$$\hat{\rho}' \sim \hat{\rho} + \lambda^2 T\|c\|_2 \frac{a}{4\pi^2} \tanh\frac{\beta\Omega}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \mathcal{O}(\lambda^4). \tag{13}$$

Note that now, when taking the limit $\Omega \to 0$, $\tanh\frac{\beta\Omega}{2} \to 0$, thus the response vanishes. There have been comments in this regard previously (Bill's), the limit of $\Omega \to 0$ is not a structureless limit. If we write the interaction Hamiltonian

$$\hat{H}_I = \lambda c(\tau)\hat{\mu}\hat{\phi}(\mathsf{x}(\tau)), \tag{14}$$

the limit $\Omega \to 0$ is a limit in which $\mu$ is time-independent. This will only resemble the accelerated charge interaction Hamiltonian, i.e.

$$\hat{H}_I = \lambda c(\tau)\hat{\phi}(\mathsf{x}(\tau)), \tag{15}$$

for particular choices of the state of the detector before the interaction. The detector itself does not distinguish between any of the infinitely many (degenerate) eigenstates of the detector free Hamiltonian, (any combination of what we called before ground state and the excited states).

However, consider that somehow (some physical implementation) we have observable access to two orthogonal states $|g\rangle, |e\rangle$ in the energy degenerate qubit Hilbert space for some reason. Consider too that we can define a (time-independent) monopole operator of the form

$$\hat{\mu} = |e\rangle\langle g| + |g\rangle\langle e|\,, \tag{16}$$

which would correspond to the limit $\Omega \to 0$ of the interaction picture monopole operator

$$\hat{\mu} = |e\rangle\langle g|\, e^{\mathrm{i}\Omega\tau} + |g\rangle\langle e|\, e^{-\mathrm{i}\Omega\tau}. \tag{17}$$

Say now that we use this Hamiltonian to describe a particle without charge. In the qubit Hilbert space there will be many states that do not commute with $\hat{\mu}$. That is states with some level of uncertainty in the monopole moment. It is only through the uncertainty of the monopole moment that we would be able to generate Larmor radiation. However, note that in this limit the radiation will not depend on the state of the field! which was our first objection (Bill's objection, actually).

It is interesting to note that in this limit the detector does not thermalize with the field, but just dephases the detector in the basis of the monopole operator we can compute fully non-perturbatively in this limit, by using that the Magnus expansion is actually summable in closed form [4].

What the zero-gap detector does in this case can be illustrated with the following image:

Build the Bloch sphere of the detector HIlbert space and pick the $z$ axis in the direction of the monopole moment (the north and south poles are $|e\rangle$ and $|g\rangle$). At the beginning, if the detector is not in the pole states, it will start rotating in the Bloch sphere around the $z$ axis, and slowly collapsing to the some point in the $z$ axis. Upon this collapse the detector is in a fixed point and the radiation emitted is constant and only depends on what height in the $z$ axsis the detector ends up. This is completely independent fo the field state, and only depend on the $z$ component of the initial state of the detector. We think it would be very intersting and one of the reasons we like Stephen and George's idea is that perhaps we could relate a full dynamical UDW reponse when accelerated to some limit of the behaviour of the detector in which we have already observed emission (for example multipolar stimulated transition emission, etc in atomic physics) because in that case the radiation emitted does indeed depend on the state of the field.

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