In what sense uniformly accelerated charges radiate for inertial observers but do not for Rindler ones.

– simplified discussion –

VERSION 2

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Abstract

We revisit the long-standing question of whether or not uniformly accelerated sources radiate for coaccelerating observers; we keep our discussion as down-to-Earth as possible to avoid unphysical issues that mystify the problem. We end up explaining in what conditions observing Larmor radiation reflects the existence of the Unruh thermal bath.

1 Main Part

1. In 1992 it was shown [1] that the emission of a usual photon from a uniformly accelerated charge in the Minkowski vacuum corresponds to either the absorption from or the emission to the Unruh thermal bath of a zero-energy Rindler photon.

2. We recall that zero-energy Rindler modes have, in general, nonzero transverse momenta and so are nontrivial. We recall in addition, that Rindler frequency and transverse momenta are independent of each other being linked by no dispersion relation.

3. Because zero-energy Rindler photons concentrate on the horizon, (physical) Rindler observers do not have any access to them. This harmonizes the fact that uniformly accelerated charges radiate for inertial observers but do not for coacelerating (Rindler) ones, who only see a distorted Coulomb field.

4. Perhaps because the concept of zero-energy Rindler photons may be unfamiliar or maybe because of the regularization procedure used there, the resolution given in [1] to the question whether or not uniformly accelerated sources radiate with respect to coaccelerating observers does not seem widely accepted yet.

5. Our primary aim here is to render the original argument given in [1] in terms of Unruh-DeWitt detectors which are more familiar to most of the people. Our secondary aim is to enumerate recent spin-offs, which hopefully will be discussed in more detail further in this workshop.

6. Cautionary remark: Rendering a piece of work to a more familiar language, many times like in poetry, leads to the impoverishment of the original message. I apologize in advance to my collaborators if they find this is what I did here. I hope this may be compensated by leading more people to visit the original papers. The paragraphs will be enumerated to make easier the discussion for the audience. Natural units are assumed $\hbar = c = k = 1$.

7. Let us begin considering the usual Unruh-DeWitt two-level pointlike detector represented by the monopole operator $\hat{m}(\tau)$. It acts on the detector energy eigenstates $|E_\pm\rangle$ as

$$\hat{m}(0)|E_\pm\rangle = |E_\mp\rangle, \quad (E_+ > E_-, \ \langle E_\pm|E_\mp\rangle = 0, \ \langle E_\pm|E_\pm\rangle = 1). \tag{1}$$

8. The detector is minimally coupled to a massless scalar field $\hat{\phi}$ through the interaction action

$$\mathcal{S}_I = \int d\tau c(\tau)\hat{m}(\tau)\hat{\phi}[x^\mu(\tau)], \tag{2}$$

where $x^\mu = x^\mu(\tau)$ is the detector worldline and τ is its proper time. The switching function $c = c(\tau)$ is assumed to be at least C^0 (to avoid unphysical divergencies [2]). The detector is kept switched on only for a finite amount of proper time T.

9. You may think of the detector as follows:

   (a) It enters the Rindler wedge inertially when it is still switched off,

   (b) Then, an external agent puts it into a uniformly accelerated trajectory with proper acceleration a,

   (c) Once the detector is uniformly accelerated, it is switched on,

   (d) The detector stays so (uniformly accelerated and switched on) for proper time T,

   (e) After this, the process is reversed back; firstly, it is switched off, and, next, it is set inertial before leaving the wedge.

10. Obviously, what the detector does while it is switched off will not affect our calculations; still, it is conceptually useful to have the whole physical picture in mind. We want to preclude our massive detector (it has internal energy; so it has mass) to asymptotically approach the speed of light: $v \to c$. A real parameter α regulates how fast the detector is switched on/off: the larger the α the faster the detector is switched on/off (i.e., the larger the α the smaller the switching time).

11. We stress that we want to keep out from this presentation the (interesting but academic) discussion about whether or not eternally accelerated electric charges (if it were possible to accelerate a charge forever) would radiate according to inertial observers themselves. We assume that we all agree that accelerated charges with physical trajectories do radiate according to the Larmor formula for inertial observers. Indeed, this is used as a benchmark in the discussion below.

12. It is much easier to calculate the excitation rate of a uniformly accelerated detector using Rindler observers since the detector is static for them; then, all calculations next are performed by these observers. (We will follow here Ref. [2].)

13. We assume that we are in the Minkowski vacuum. Then, Rindler observers must use the fact that they are in a thermal state with the Unruh temperature [3].

14. According to the action (2), the detector can excite by absorbing or emitting a Rindler particle with Rindler frequency ω_R and transverse momentum $k_\perp$. (In the latter case, the external agent switching on/off the detector is the one who provides the energy to allow such a process.)

15. The detector excitation probability with simultaneous absorption of a Rindler particle from the Unruh thermal bath is

$$\mathcal{P}^{exc}_{abs} = \int dW^{exc}_{abs} \frac{1}{e^{\omega_R/T_U} - 1}, \tag{3}$$

where $T_U = a/(2\pi)$,

$$dW^{exc}_{abs} \equiv |\mathcal{A}^{exc}_{abs}|^2 d^2\mathbf{k}_\perp d\omega_R, \tag{4}$$

and

$$\mathcal{A}^{exc}_{abs} = \langle 0_R| \otimes \langle E_+|\mathcal{S}_I|E_-\rangle \otimes |\omega_R \mathbf{k}_\perp\rangle, \tag{5}$$

where $|0_R\rangle$ is the Rindler vacuum.

16. Analogoulsly, the excitation probability with simultaneous emission of a Rindler particle into the Unruh thermal bath is given by

$$\mathcal{P}^{exc}_{em} = \int dW^{exc}_{em} \left(1 + \frac{1}{e^{\omega_R/T_U} - 1}\right), \tag{6}$$

where

$$dW^{exc}_{em} = |\mathcal{A}^{exc}_{em}|^2 d^2\mathbf{k}_\perp d\omega_R. \tag{7}$$

and

$$\mathcal{A}^{exc}_{em} = \langle \omega_R \mathbf{k}_\perp | \otimes \langle E_+ | \, \mathcal{S}_I \, | E_- \rangle \otimes | 0_R \rangle. \tag{8}$$

The two terms in the parentheses in Eq. (6) are associated with spontaneous and induced emissions, respectively.

17. Thus, according to Rindler-observers' calculations, the total excitation probability will be

$$\mathcal{P}^{exc} = \mathcal{P}^{exc}_{em} + \mathcal{P}^{exc}_{abs}. \tag{9}$$

This is also precisely what inertial observers measure since the excitation probability of a detector is an observable which depends only on the detector properties and field state. This can be explicitly verified by a direct inertial frame calculation.

18. So far so good. However, we are interested in radiation emission rather than on detector excitation. It lacks, thus, to calculate the emission and absorption of Rindler particles associated with *detector deexcitation.* If we aim to look at radiation emission, we must eventually trace out on the detector internal state. The corresponding results are expressed below.

19. Deexcitation probability with simultaneous absorption of a Rindler particle from the Unruh thermal bath:

$$\mathcal{P}^{deexc}_{abs} = \int dW^{deexc}_{abs} \frac{1}{e^{\omega_R/T_U} - 1}, \tag{10}$$

where

$$dW^{deexc}_{abs} \equiv |\mathcal{A}^{deexc}_{abs}|^2 d^2\mathbf{k}_\perp d\omega_R, \tag{11}$$

with

$$\mathcal{A}^{deexc}_{abs} = \langle 0_R| \otimes \langle E_-|\mathcal{S}_I|E_+\rangle \otimes |\omega_R \mathbf{k}_\perp\rangle \tag{12}$$

20. Deexcitation probability with simultaneous emission of a Rindler particle into the Unruh thermal bath

$$\mathcal{P}^{deexc}_{em} = \int dW^{deexc}_{em} \left(1 + \frac{1}{e^{\omega_R/T_U} - 1}\right), \tag{13}$$

where

$$dW^{deexc}_{em} = |\mathcal{A}^{deexc}_{em}|^2 d^2\mathbf{k}_\perp d\omega_R. \tag{14}$$

with

$$\mathcal{A}^{deexc}_{em} = \langle \omega_R \mathbf{k}_\perp| \otimes \langle E_-|\ \mathcal{S}_I\ |E_+\rangle \otimes |0_R\rangle. \tag{15}$$

21. The switching on/off process was only introduced to keep our problem as physical as possible. Notwithstanding, we do not want our conceptual message to depend on T or α. Hence, we will focus on the regime where (i) the detector stays switched on for a time interval T larger than any other time scale in the problem and (ii) the time scales defined by a^{-1} and ΔE^{-1} are both large compared with the switching on/off time scale α^{-1} (keeping in mind that the switching on/off cannot be instantaneous in order to avoid ultraviolet divergencies):

$$T \gg (a^{-1}, \Delta E^{-1}) \gg \alpha^{-1} > 0, \tag{16}$$

where $\Delta E \equiv E_+ - E_- > 0$.

22. In the regime (16), the leading term for the transition rates

$$\Gamma^{exc/deexc}_{em/abs} \equiv P^{exc/deexc}_{em/abs}/T$$

will only depend on a and ΔE (see, e.g., Ref. [2] for more details).

23. One of our aims is to show that a detector with unresolved inner-structure radiates as a structureless scalar source does. If the inner structure is unresolved, there is no way to know whether it is excited or unexcited. We cope with it considering the detector's initial state to be a mixed state $\hat{\rho} = a|E_-\rangle\langle E_-| + b|E_+\rangle\langle E_+|$. For the sake of simplicity, we consider that it is initially half excited and half deexcited (but any mixed state would not alter the final result (21)). Then, the combined absorption and emission rates (associated with either excitation or deexcitation) can be cast in the regime (16) as

$$\Gamma^L \equiv (\Gamma^{exc}_{em} + \Gamma^{deexc}_{em} + \Gamma^{exc}_{abs} + \Gamma^{deexc}_{abs})/2 \tag{17}$$

$$\approx (\Gamma^{deexc}_{em} + \Gamma^{exc}_{abs})/2, \tag{18}$$

where the 1/2 factor appears because we are averaging on initial excited/unexcited detector states. We note that Γ^{exc}_{em} and Γ^{deexc}_{abs} were dismissed because in the regime (16), we have $\mathcal{A}^{exc}_{em}, \mathcal{A}^{deexc}_{abs} \underset{\sim}{\propto} \delta(\omega_R + \Delta E)$ – reflecting energy conservation.

24. Now, a straightforward calculation allows us to cast Γ^L as

$$\Gamma^L \approx \frac{c_0^2}{2}\left[\frac{\Delta E}{2\pi} + 2 \times \frac{\Delta E}{2\pi}\frac{1}{e^{\Delta E/T_U} - 1}\right]\int_0^\infty d\omega_R \delta[\omega_R - \Delta E], \tag{19}$$

where $c_0 = \text{const}$ was "hidden" in $c(\tau)$ and should be interpreted as the coupling constant between the detector and field.

25. The integral is maintained to make explicit that in the regime (16) energy conservation only allows the detector to interact with particles of the thermal bath with $\omega_R = \Delta E$.

26. Now, suppose that the energy gap is small with respect to the Unruh thermal bath, $\Delta E \ll T_U = a/(2\pi)$. In this case Eq. (19) reduces to

$$\Gamma^L|_{\Delta E \ll a} \approx \frac{c_0^2}{2\pi}\frac{\Delta E}{e^{2\pi\Delta E/a} - 1}\int_0^\infty d\omega_R \delta[\omega_R - \Delta E]. \tag{20}$$

Clearly, such a detector is only able to interact with infrared Rindler particles $\omega_R = \Delta E \ll a$.

27. Think, now, of the physical situation where ΔE is so small that the detector inner structure cannot be resolved by current technology. Then, by (i) performing the integral in Eq. (20) and next (ii) taking the $\Delta E/a \to 0$ limit, we have

$$\Gamma^L|_{\Delta E/a \to 0} \approx \frac{c_0^2 a}{4\pi^2}, \tag{21}$$

where $\approx$ is only used because the detector is not accelerated forever. This is, thus, the combined absorption and emission rates of super infrared ("zero-energy") Rindler particles associated with our detector with an unresolved inner structure.

28. Because each *absorption* or *emission* of a Rindler particle uniquely corresponds to the *emission* of a Minkowski particle [4], Eq. (21) is also precisely the emission rate of usual Minkowski particles radiated by our detector (with unresolved inner structure) as seen by inertial observers. This can be verified through a usual inertial-frame calculation.

29. We call attention to the fact, now, that a straightforward inertial-frame calculation for the emission rate of Minkowski particles radiated by a *structureless* pointlike uniformly accelerated classical scalar source $j(x)$ coincides with Eq. (21) [5]:

$$\Gamma^L|_{\Delta E=0} = \frac{c_0^2 a}{4\pi^2}, \tag{22}$$

i.e., *the emission of a Minkowski particle from a UD detector with unresolved inner structure coincides with the one from a structureless scalar source and corresponds to the emission/absorption of a zero-energy Rindler particle to/from the Unruh thermal bath* – recall the limit $\Delta E/a \to 0$ in par 27.

30. If one compares the present approach with Ren and Weinberg's one [5], one will see that what we have done here is to replace Ren and Weinberg's oscillating scalar source by the UD detector (no dipoles, let me emphasize). Our $\Delta E \to 0$ final limit corresponds in their work to the procedure of vanishing the oscillation at the end.

31. Translating this to electric charges one can say that [1]: *the emission of a usual (finite-energy) Minowski photon from a uniformly accelerated charge as defined by*

inertial observers corresponds to either the absorption or emission of a zero-energy Rindler photon as defined by Rindler observers. (The same conclusion can be obtained from Ref. [8], where no oscillating dipoles are introduced.)

32. Now it is easy to understand why inertial observers do see radiation from a uniformly accelerated source, while Rindler observers do not. One way to understand it is to recall that zero-energy Rindler modes concentrate on the horizon, where physical Rindler observers do not have access.

33. Recently the relationship above between zero-energy Rindler particles and finite-energy Minkowski particles was strengthened even more by explicitly showing how *the usual classical Larmor radiation can be entirely built from the zero-energy Rindler modes* [6].

34. Because zero-energy Rindler modes present in the Unruh thermal bath are crucially used to build Larmor radiation, we claim that **under proper conditions** the observation of Larmor radiation is circumstantial evidence for the Unruh thermal bath. The link between the classical Larmor radiation and the quantum Unruh effect requires the use of a simple extra quantum ingredient, namely the Planck-Einstein relation $E = h\nu$ [7].

35. The proper conditions we refer above are discussed in [8]-[9] and are mostly related to the fact that the charge must have enough time to thermalize in the Unruh thermal bath. Although the experiment is feasible under present technology, it was not realized yet to the best of our knowledge.

36. Larmor radiation may be seen as a shadow of the Unruh thermal bath in Plato's cave. It reveals some thing about the true thing but not the whole thing. Anyway, is not that always the case?

2 Appendix – Degenerated UD detector

37. Now, let us see what happens if the inner structure of the detector is collapsed from the start. By collapsed, we mean that $E_+ = E_- \equiv E$, in which case (see Eq. (1))

$$\hat{m}(0)|E\rangle = |E\rangle, \quad \langle E|E\rangle = 1. \tag{23}$$

38. Once, this is done, it becomes pointless to talk of excitation and deexcitation and par 23 must be changed accordingly. The combined emission and absorption rates are then cast as

$$\begin{aligned} \Gamma^L &\equiv \Gamma_{em} + \Gamma_{abs} &(24)\\ &\equiv (\mathcal{P}_{em} + \mathcal{P}_{abs})/T, &(25)\\ &= \int \frac{dW_{em}}{T}\left[1 + \frac{1}{e^{\omega_R/T_U} - 1}\right] + \int \frac{dW_{abs}}{T}\frac{1}{e^{\omega_R/T_U} - 1} &(26) \end{aligned}$$

with

$$dW_{em/abs} \equiv |\mathcal{A}_{em/abs}|^2 d^2\mathbf{k}_\perp d\omega_R, \tag{27}$$

and

$$\begin{aligned} \mathcal{A}_{em} &= \langle \omega_R \mathbf{k}_\perp| \otimes \langle E|\ \mathcal{S}_I\ |E\rangle \otimes |0_R\rangle. &(28)\\ \mathcal{A}_{abs} &= \langle 0_R| \otimes \langle E|\mathcal{S}_I|E\rangle \otimes |\omega_R \mathbf{k}_\perp\rangle. &(29) \end{aligned}$$

39. This can be evaluated as before, leading to

$$\Gamma^L \approx c_0^2 \int_0^\infty d\omega_R \left[\frac{\omega_R}{2\pi} + 2\times\frac{\omega_R}{2\pi}\frac{1}{e^{\omega_R/T_U} - 1}\right]\delta(\omega_R). \tag{30}$$

Compare it with Eq.(19) in par 24 (and recall that there is no averaging here to understand why we have here c_0^2 rather than $c_0^2/2$).

40. Now, in order to make sense of it, we must cast it as (this is the price of dealing with a degenerate pair of modes from the start)

$$\lim_{\omega_R\to 0}\left[\frac{\omega_R}{2\pi} + 2\times\frac{\omega_R}{2\pi}\frac{1}{e^{\omega_R/T_U} - 1}\right] = \frac{a}{2\pi^2} \tag{31}$$

and

$$\int_0^\infty d\omega_R \delta(\omega_R) = 1/2. \tag{32}$$

Using it in Eq. (30), we recover again Eq. (21):

$$\Gamma^L \approx \frac{c_0^2 a}{4\pi^2}. \tag{33}$$

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