The mass of the matter in a cube centered at
\[ \mathbf{r} = \langle x, y, z \rangle = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \]
is the sum of the masses of the particles in the cube. The average density in the cube is its mass over its volume:
\[ \frac{M_{\text{cube}}}{\Delta x \Delta y \Delta z} = \frac{M_{\text{cube}}}{(\Delta x)^3}. \]
Take the limit as the cube shrinks to the point \( \mathbf{r} \):
\[ \rho(\mathbf{r}) = \lim_{\Delta x \to 0} \frac{M_{\text{cube at } \mathbf{r}}}{(\Delta x)^3}. \]
This is the density at \( \mathbf{r} \).

Now we think of the total mass as the sum of the masses in all the cubes. In the limit of infinitesimal cubes it’s
\[ M = \iiint_E \rho(\mathbf{r}) \, dV = \iiint_E \rho(\mathbf{r}) \, dx \, dy \, dz, \]
where \( E \) is the region in 3-space occupied by the body.

[Ultimately should insert a link, “For more details click here”, leading to a Riemann-sum discussion of this integral.]