Taylor’s Theorem, Version 1

If all the derivatives of the function $f$ up through $f^{(N+1)}$ exist in an interval $I$ containing the number $a$, then for all $x$ in $I$, $f(x)$ is well approximated by its $N$th-degree Taylor polynomial,

$$T_N(x) = \sum_{j=0}^{N} \frac{f^{(j)}(a)}{j!}(x - a)^j,$$

in the following sense:

$$f(x) = T_N(x) + R_N(x),$$

where

$$|R_N(x)| \leq \frac{M|x - a|^{N+1}}{(N + 1)!}$$

with

$$M = \max_{z \in I} |f^{(N+1)}(z)|.$$