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## The Greek Alphabet

| | | | | |
|---|---|---|---|---|
| A | ..... | $\alpha$ | ..... | alpha |
| B | ..... | $\beta$ | ..... | beta |
| $\Gamma$ | ..... | $\gamma$ | ..... | gamma |
| $\Delta$ | ..... | $\delta$ | ..... | delta |
| E | ..... | $\epsilon$ | ..... | epsilon |
| Z | ..... | $\zeta$ | ..... | zeta |
| H | ..... | $\eta$ | ..... | eta |
| $\Theta$ | ..... | $\theta$ | ..... | theta |
| I | ..... | $\iota$ | ..... | iota |
| K | ..... | $\kappa$ | ..... | kappa |
| $\Lambda$ | ..... | $\lambda$ | ..... | lambda |
| M | ..... | $\mu$ | ..... | mu |
| N | ..... | $\nu$ | ..... | nu |
| $\Xi$ | ..... | $\xi$ | ..... | xi |
| O | ..... | o | ..... | omicron |
| $\Pi$ | ..... | $\pi$ | ..... | pi |
| P | ..... | $\rho$ | ..... | rho |
| $\Sigma$ | ..... | $\sigma$ | ..... | sigma |
| T | ..... | $\tau$ | ..... | tau |
| $\Upsilon$ | ..... | $\upsilon$ | ..... | upsilon |
| $\Phi$ | ..... | $\phi$ | ..... | phi |
| X | ..... | $\chi$ | ..... | chi |
| $\Psi$ | ..... | $\psi$ | ..... | psi |
| $\Omega$ | ..... | $\omega$ | ..... | omega |