HW 10

① Describe all ring homomorphisms of

    a) $\mathbb{Z}$ into $\mathbb{Z}$

    b) $\mathbb{Z}$ into $\mathbb{Z} \times \mathbb{Z}$

    c) $\mathbb{Z} \times \mathbb{Z}$ into $\mathbb{Z}$.

② Find all solutions of the equation

$x^2 + x - 6 = 0$ in the ring $\mathbb{Z}_{14}$ by factoring the quadratic polynomial.

③ Find all solutions of the equation

    a) $x^2 + 2x + 4 = 0$ in $\mathbb{Z}_6$.

    b) $x^3 - 2x^2 - 3x = 0$ in $\mathbb{Z}_{12}$.

④ Find the characteristic of the given ring

    a) $\mathbb{Z}_5 \times \mathbb{Z}_5$ b) $7\mathbb{Z}$ c) $\mathbb{Z}_5 \times \mathbb{Z}_8$

d) $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$, e) $\mathbb{Z}_{11} \times 5\mathbb{Z}$.

⑤ Let R be a commutative ring with unity of characteristic 3. Compute and simplify $(a+b)^9$ for $a, b \in R$.

⑥ Show that the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is a divisor of zero in $M_2(\mathbb{Z})$.

⑦ Answer each of the questions from Problem 17 in Exercises 19 of the book. Justify your answers or give a reference to the corresponding statement in the book.

⑧ Use Fermat's theorem to find the remainder of 37^{49} when it is divided by 7.

⑨ Compute $\varphi(49)$, where φ is the Euler phi-function.

(10) Describe all solutions of the given congruence

a) $36x \equiv 15 \pmod{24}$

b) $36x \equiv 12 \pmod{24}$.