HW 12.

(1) Let D be an integral domain and x an indeterminate.

a) Describe the units in $D[x]$.

b) Find the units in $\mathbb{Z}[x]$.

c) Find the units in $\mathbb{Z}_7[x]$.

(2) Find $q(x)$ and $r(x)$ as described by the division algorithm so that $f(x) = g(x)q(x) + r(x)$ with $r(x) = 0$ or of degree less than the degree of $g(x)$

a) $f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2$ and $g(x) = 3x^2 + 2x - 3$ in $\mathbb{Z}_7[x]$.

b) $f(x) = x^5 - 2x^4 + 3x - 5$ and $g(x) = 2x + 1$ in $\mathbb{Z}_{11}[x]$

(3) Find all generators of the cyclic multiplicative group of units of the given finite field.

a) $\mathbb{Z}_7$ b) $\mathbb{Z}_{23}$

(4) The polynomial x^3+2x^2+2x+1 can be factored into linear factors in $\mathbb{Z}_7[x]$. Find this factorization.

(5) is x^3+2x+3 an irreducible polynomial in $\mathbb{Z}_5[x]$? Why? Express it as a product of irreducible polynomials of $\mathbb{Z}_5[x]$.

(6) Show that $f(x)=x^2+8x-2$ is irreducible over $\mathbb{Q}$. is $f(x)$ irreducible over $\mathbb{R}$? Over $\mathbb{C}$?

(7) Demonstrate that x^4-22x^2+1 is irreducible over $\mathbb{Q}$.

(8) Answer each of the questions from Problem 25 in Exercises 23 of the book. Justify your answers or give a reference to the corresponding statement in the book.