

HW 13

① Demonstrate that $x^4 - 22x^2 + 1$ is irreducible over \mathbb{Q} .

② Determine whether the polynomial in $\mathbb{Z}[x]$ satisfies an Eisenstein criterion

a) $x^2 - 24$, b) $4x^{10} - 9x^3 + 24x - 18$

③ Find all irreducible polynomials

a) of degree 3 in $\mathbb{Z}_2[x]$.

b) of degree 3 in $\mathbb{Z}_3[x]$.

④ Show that for p a prime, the polynomial $x^p + a$ in $\mathbb{Z}_p[x]$ is not irreducible for any $a \in \mathbb{Z}_p$.

⑤ if F is a field and $a \neq 0$ is a zero of $f(x) = a_0 + a_1x + \dots + a_nx^n$ in $F[x]$,

show that $\frac{1}{a}$ is a zero of $a_n + a_{n-1}x + \dots + a_0x^n$.

(6) Find the number of irreducible quadratic polynomials in $\mathbb{Z}_p[x]$, where p is a prime. [Hint: Find the number of reducible polynomials of the form $x^2 + ax + b$, then the number of reducible quadratics, and subtract this from the total number of quadratics].