HW 3.

(1) Find the quotient and remainder, according to the division algorithm, when n is divided by m.

(a) $n = 51$, $m = 7$

(b) $n = -51$, $m = 7$.

(2) Find the greatest common divisor of the two integers

(a) 16 and 24

(b) 21 and 48

(c) $2^2 \cdot 3^2 \cdot 5 \cdot 7^3$ and $2 \cdot 3^3 \cdot 5^2 \cdot 7$

(3) Find the number of generators of a cyclic group having the given order.

(a) 17, (b) 20 (c) 24

④ An isomorphism of a group with itself is an automorphism of the group. Find the number of automorphisms of the given group

(a) $\mathbb{Z}_3$ (b) $\mathbb{Z}_{15}$ (c) $\mathbb{Z}_{24}$.

⑤ Find the number of elements in the indicated cyclic group.

(a) The cyclic subgroup of $\mathbb{Z}_{35}$ generated by

(a) 5 (b) 7, (c) 3, d) 10.

⑥ Find all subgroups of the given group and draw the subgroup diagram for the subgroups

(a) $\mathbb{Z}_{15}$ (b) $\mathbb{Z}_{30}$, (c) $\mathbb{Z}_{27}$.

Find orders of all subgroups in each case.

(7) Answer each of the questions from Problem 32 in Exercises 6 of the Fraleigh book. Justify your answers or give a reference to the corresponding statement in the book.

(8) The generators of the cyclic multiplicative group U_n of all nth roots of unity in $\mathbb{C}$ are the primitive nth roots of unity. Find the primitive n-th roots of unity for the given value of n.

(a) $n = 15$ (b) $n = 27$.

(9) Compute the indicated product involving the following permutations in S_6:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}, \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$$

$$\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$$

[i] (a) $\sigma\tau$, (b) $\sigma\tau^2$ c) $\mu^{-1}\sigma^2$

[ii] Compute the orders of elements σ, τ, μ, $\sigma\tau$, $\sigma\tau^2$, $\mu^{-1}\sigma^2$.

[iii] Compute τ^{99}, μ^{200}.

(10) Answer each of the questions from Problem 35 in Exercises 8 of the Fraleigh book. Justify your answers or give a reference to the corresponding statement in the book.