HW 4.

① $\boxed{\text{i}}$ Find all orbits of the given permutations:

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 2 & 1 & 6 & 5 & 4 \end{pmatrix} = \sigma$

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 5 & 1 & 2 & 8 & 6 & 7 \end{pmatrix} = \tau$

$\boxed{\text{ii}}$ Find orders of σ and τ.

$\boxed{\text{iii}}$ Express σ and τ as a product of disjoint cycles, and then as a product of transpositions.

② What is order in S_8 of the

(a) cycle $(1, 7, 3, 2)$

(b) $\sigma = (3, 2)(1, 5, 7)$

(c) $\tau = (1, 3)(2, 7)(3, 5, 6)$?

(3) Find the maximum possible order for an element of S_n for the given values of n.

(a) $n = 4$ (b) $n = 8$, (c) $n = 12$.

(4) Answer each of the questions from Problem 23 in Exercises 9 of the Fraleigh's Book. Justify your answers or give a reference to the corresponding statement in the Book.

(5) Let τ be the 8-cycle $(1,2,3,4,5,6,7,8)$. For which positive integers i is τ^i also an 8-cycle?

(6) Find all cosets of the subgroup:

(a) $5\mathbb{Z}$ of $\mathbb{Z}$, (b) $15\mathbb{Z}$ of $30\mathbb{Z}$.

(c) $\{\rho_0, \mu_1\}$ of the group D_4 given by the table 8.12.

(d) Find the right cosets for (a), (b), (c). Are they the same as the left cosets?

(6) Find the index

(a) of $<5>$ in the group $\mathbb{Z}_{60}$

(b) of $<\mu_2>$ in the group S_3, using the notation of Example 10.7

(c) of $<\mu_2>$ in the group D_4 given in Table 8.12.

(7) Answer each of the questions from Problem 19 in Exercises 10 of the Fraleigh's book. Justify your answers or give

a reference to the corresponding statement in the Book.

(10) Prove that the relation $\sim_R$ of Theorem 10.1 is an equivalence relation.

(11) Show that a group with at least two elements but with no proper nontrivial subgroups must be finite and of prime order.