HW5.

(1) (Problem 7 from HW4). Answer each of the questions from Problem 19 in Exercises 10 of the Fraleigh's Book. Justify your answers or give a reference to the corresponding statement in the Book.

(2) Find the index

(a) of $<\rho_1>$ in the group S_3, using the notation of Example 10.7 in the Book.

(b) of $<\delta_2>$ in the group D_4 given in the Table 8.12

(c) of $<\tau>$ in S_5 where $\tau = (1,3,5)(3,2)$

(d) of $<\eta>$ in S_6 where $\eta = (3,6)(1,4,2,5)$.

(3) Let H and K be subgroups of a group G. Define $\sim$ on G by $a \sim b$ if and only if $a = hbk$ for some $h \in H$ and some $k \in K$.

(a) Prove that $\sim$ is an equivalence relation on G.

(b). Describe the elements in the equivalence class containing $a \in G$. (These equivalence classes are called <u>double cosets</u>).

(4) List the elements of $\mathbb{Z}_3 \times \mathbb{Z}_9$. Find the order of each of the element, is this group cyclic?

(5) Find the order of the given element in the direct product.

(a) $(3,4)$ in $\mathbb{Z}_4 \times \mathbb{Z}_{12}$.

(b) $(3,5)$ in $\mathbb{Z}_6 \times \mathbb{Z}_{15}$

(c) $(9,10,3)$ in $\mathbb{Z}_{15} \times \mathbb{Z}_{12} \times \mathbb{Z}_4$

(6) What is the largest order among the orders of all cyclic subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_8$? of $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_5$.

(7) Find all subgroups of $\mathbb{Z}_3 \times \mathbb{Z}_9$ of order 9.

(8) Find the maximal possible order for some element of $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_3$.

(9) Are the groups $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_{12}$ isomorphic? Why or why not?

(10) Find the maximum possible order for some element of $\mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_6 \times \mathbb{Z}_{15}$.

(11) Find all abelian groups, up to isomorphism, of the given order.

(a) Order 125

(b) Order 64

(c) Order $2^3 \cdot 3^2 \cdot 5^2$.