

HW 7.

(1) How many homomorphisms are there of

a) \mathbb{Z} into \mathbb{Z}_6 ,

b) \mathbb{Z} into \mathbb{Z} ,

c) \mathbb{Z} onto \mathbb{Z} . ?

(2) Give an example of a nontrivial homomorphism φ for the given groups, if an example exists. If no such homomorphism exists, explain why that is so.

a) $\varphi: \mathbb{Z}_{15} \rightarrow \mathbb{Z}_4$

b) $\varphi: \mathbb{Z}_5 \rightarrow S_5$

c) $\varphi: \mathbb{Z}_3 \times \mathbb{Z}_9 \rightarrow \mathbb{Z}_7 \times \mathbb{Z}_3$

d) $\varphi: \mathbb{Z}_{11} \rightarrow \mathbb{Z}$

e) $\varphi: 5\mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$

f) $\varphi: \mathbb{Z} \times \mathbb{Z} \rightarrow 23\mathbb{Z}$.

(3) Answer each of the questions from Problem 32 in Exercises 13 of the book. Justify your answers or give a reference to the corresponding statement in the book.

(4) Show that any group homomorphism $\varphi: G \rightarrow G'$ where $|G|$ is a prime must either be the trivial homomorphism or a one-to-one map.

(5) Find the order of the given factor group

a) $\mathbb{Z}_{15} / \langle 3 \rangle$

b) $(\mathbb{Z}_9 \times \mathbb{Z}_2) / \langle (3, 1) \rangle$

c) $(\mathbb{Z}_3 \times S_3) / \langle (1, s_2) \rangle$

d) $(\mathbb{Z}_{15} \times \mathbb{Z}_{18}) / \langle (3, 6) \rangle$

(6) Give the order of the element in the factor group

a) $7 + \langle 3 \rangle$ in $\mathbb{Z}_{15} / \langle 3 \rangle$

b) $(2, 1) + \langle (1, 1) \rangle$ in $(\mathbb{Z}_5 \times \mathbb{Z}_4) / \langle (1, 1) \rangle$