

HW 9.

(1) Find both the center and the commutator subgroup

a) of D_4

b) of $\mathbb{Z}_5 \times S_3$

(2) Answer each of the questions from Problem 19 in Exercises 15 of the book. Justify your answers or give a reference to the corresponding statement in the book.

(3) Let $\varphi: G \rightarrow G'$ be a group homomorphism.

a) Show that if N is a normal subgroup of G , then $\varphi[N]$ is normal subgroup of $\varphi[G]$.

b) Show that if N' is a normal subgroup of G' , then $\varphi^{-1}[N']$ is a normal subgroup of G .

(4) Show that if G is nonabelian, then the factor group $G/Z(G)$ is not cyclic.

[Hint: Show the equivalent contrapositive, namely, that if $G/Z(G)$ is cyclic then G is abelian (and hence $Z(G) = G$).]

(5) Using previous Problem, show that a nonabelian group G of order pq where p and q are primes has a trivial center.

(6) Answer each of the questions from Problem 33 in Exercises ~~33~~¹⁸ of the book.

Justify your answers or give a reference to the corresponding statement in the book.

(7) Compute the product in the given ring.

a) $(16)(12)$ in \mathbb{Z}_{24} , b) $(-4)(11)$ in \mathbb{Z}_5

c) $(2, 4)(4, 7)$ in $\mathbb{Z}_5 \times \mathbb{Z}_9$

(8) Solve Problems 7-11 from Exercises 18 of the book.

(9) Describe all units in the given ring

a) \mathbb{Z}_7 , b) \mathbb{Z}_8 c) \mathbb{Z} , d) \mathbb{Q}

e) $\mathbb{Z} \times \mathbb{Z}$ f) $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Q}$.