The theme of these lectures will be three interrelated topics in metric geometry: bi-Lipschitz (non)embeddability results, extension theorems for Lipschitz functions, and quantitative versions of differentiation theorems. The aim will be to explain recent advances in these directions, explain how they are related to the above themes, explain connections to asymptotic convex geometry, isoperimetry and algorithms, and present important open problems. Examples of topics to be covered include sharp nonembeddability of the Heisenberg group into $L_1$ and (consequently) the sharp resolution of the Goemans-Linial conjecture (joint work with Robert Young), improved extension theorems from subsets of $\ell_p$ spaces, improved non-extendability lower bounds and their relation to expanders and transportation cost (joint works with Manor Mendel and Yuval Rabani), and quantitative differentiation theorems for uniformly convex targets (joint work with Tuomas Hytönen).