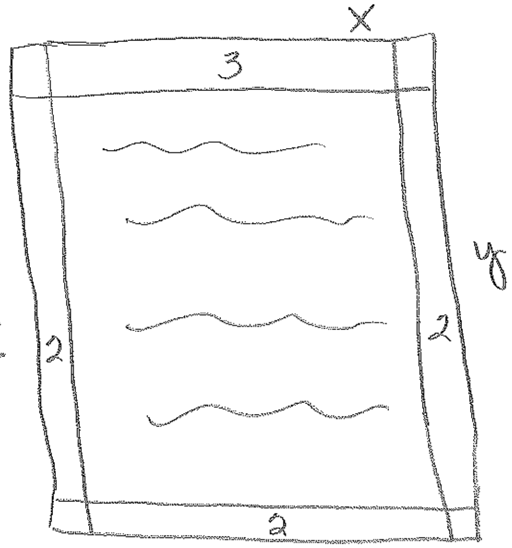
Week - In - Review Nov. 12: Section 5.4 - Section 5.5

1. A poster is to have 240 sq. in. with 2-in margins at the bottom and the sides and a 3-in margin at the top. What dimensions will give the largest printed area?

Domain: $x, y > 0$

$x \cdot y = 240 \Rightarrow y = \frac{240}{x}$



$y = \frac{240}{\sqrt{192}}$

$x = \sqrt{192}$

$$\begin{pmatrix}\text{Printed}\\ \text{Area}\end{pmatrix} = (y-5)(x-4)$$

$$= xy - 5x - 4y + 20$$

$$= x\left(\frac{240}{x}\right) - 5x - 4\left(\frac{240}{x}\right) + 20$$

$$= 240 - 5x - \frac{960}{x} + 20$$

$$= 260 - 5x - \frac{960}{x}$$

$$\begin{pmatrix}\text{Printed}\\ \text{Area}\end{pmatrix}' = -5 + \frac{960}{x^2}$$

$$= \frac{960 - 5x^2}{x^2}$$

$960 - 5x^2 = 0$

$5(192 - x^2) = 0$

$x = \sqrt{192}$

Note: $x \neq 0$, $x \neq -\sqrt{192}$

$$\begin{pmatrix}\text{Printed}\\ \text{Area}\end{pmatrix}'' = -\frac{960(2)}{x^3} \quad (-) \Rightarrow \text{MAX} \checkmark$$

$y = \frac{240}{\sqrt{192}}$

2. Find two nonnegative numbers whose sum is 20 and whose sum of squares is maximum?

$x = 1^{st}$ #

$20 - x = 2^{ND}$ #

$x + (20 - x) = 20$

Maximize $= x^2 + (20-x)^2$

Domain: $[0, 20]$

$M' = 2x + 2(20-x)(-1)$

$= 2x - 40 + 2x$

$= 4x - 40 = 0$

$4x = 40$

$x = 10$

$M'' = 4 \ (+) \Rightarrow$ minimum X

check end pts.

$M(0) = 400$

$M(10) = 200$

$M(20) = 400$

Numbers are $0 + 20$

3. Find two numbers whose difference is 100 and whose product is a minimum.

$x = 1^{st}\ \#$

$y = 2^{ND}\ \#$

$P = x \cdot y$

$= x(x-100) = x^2 - 100x$

$x - y = 100$

$y = x - 100$

$P' = 2x - 100$ undefined nowhere

$2x - 100 = 0$

$x = 50$

$P'' = 2 > 0 \Rightarrow$ min ✓

$y = 50 - 100$

$= -50$

$50\ \&\ -50$

4. A farmer wants to construct a rectangular pen next to a barn 60 ft. long, using all of the barn as part of one side of the pen. Find the dimensions of the pen with the largest area that the farmer can build if 160 ft. of fencing material is available.

$A = (60 + x)\,y = (60 + x)(50 - x)$

$= 3000 - 10x - x^2$

$A' = -10 - 2x = 0$ undef. nowhere

$x + 2y + (60 + x) = 160$

$2x + 2y = 100$

$x + y = 50$

$y = 50 - x$

$-2x = 10$

$x = -5$ nope!

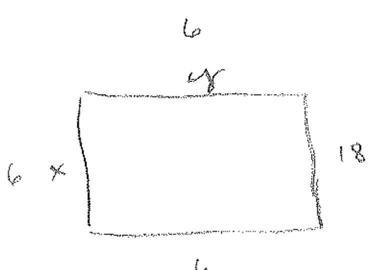
Domain: $[0, 50]$

$A(0) = 3000$

$A(50) = 0$

pen is $60' \times 50'$

5. A fence is built to enclose a rectangular area of 800 square feet. The fence along three sides is made of material that costs \$6 per foot. The material for the fourth side costs \$18 per foot. Find the dimensions of the rectangle that will allow the most economical fence to be built.



$P = 24x + 12y$

$= 24x + 12\left(\frac{800}{x}\right)$

$= 24x + \frac{9600}{x}$

$20' \times 40'$

Domain: $\mathbb{R}^+$

$P' = 24 - \frac{9600}{x^2} = \frac{24x^2 - 9600}{x^2}$

$A = x \cdot y = 800$

$y = \frac{800}{x} = \frac{800}{20} = 40$

$x = 0$ makes no sense

$24x^2 - 9600 = 0$

$x^2 - 400 = 0$

$(x+20)(x-20) = 0$

$x = \pm 20$

$x = -20$ makes no sense

$P'' = \frac{2(9600)}{x^3} > 0 \Rightarrow$ min ✓

$x = 20$ is only c.v.

6. A walnut grower estimates from past records that if 20 trees are planted per acre, each tree will average 60 pounds of nuts per year. If for each additional tree planted per acre (up to 15) the average yield per tree drops 2 pounds, how many trees should be planted to maximize the yield per acre? What is the maximum yield?

x = # of add. trees

$20 + x$ = # of trees

$60 - 2x$ = yield/tree

Domain: $[0, 15]$

$$Y(x) = (20 + x)(60 - 2x)$$
$$= 1200 + 20x - 2x^2$$

$$Y'(x) = 20 - 4x = 0$$
$$x = 5 \quad \text{nowhere undef.}$$

$$Y''(x) = -4 < 0 \Rightarrow \text{MAX} \checkmark$$

trees = 25

$$\text{Max } Y(25) = (25)(50) = 1250 \text{ lbs/acre}$$

7. A car rental agency rents 200 cars per day at a rate of $30 per day. For each $1 increase in rate, 5 fewer cars are rented. At what rate should the cars be rented to produce the maximum income?

x = \$1↑ in rate

$200 - 5x$ = # of cars rented

$30 + x$ = rate

Domain: $200 - 5x = 0$

$200 = 5x$

$x = 40$

$[0, 40]$

$$I(x) = (200 - 5x)(30 + x)$$
$$= 6000 + 50x - 5x^2$$

$$I'(x) = 50 - 10x = 0$$
$$x = 5 \quad \text{undef. nowhere}$$

$$I'' = -10 < 0 \Rightarrow \text{MAX} \checkmark$$

Rate = \$35

8. Compute the following limits:

(a) $\lim_{x \to -2} \dfrac{x^3 - x^2 - x + 10}{x^2 + 3x + 2}$ $\left(\frac{0}{0}\right)$

$$= \lim_{x \to -2} \frac{3x^2 - 2x - 1}{2x + 3} = -15$$

(b) $\lim_{x \to 0} \dfrac{1 - \cos(x)}{2x^2}$ $\left(\frac{0}{0}\right)$

$$= \lim_{x \to 0} \frac{\sin x}{4x} = \frac{1}{4} \lim_{x \to 0} \cancel{\frac{\sin x}{x}}^{\,1} = \frac{1}{4}$$

OR b/c still (0/0)

$$= \lim_{x \to 0} \frac{\cos x}{4} = \frac{1}{4}$$

(c) $\lim_{x \to \infty} \left(1 + e^{2x}\right)^{1/x}$ (∞^0)

change to

$$\lim_{x \to \infty} \frac{1}{x} \ln\left(1 + e^{2x}\right) = \lim_{x \to \infty} \frac{\ln\left(1 + e^{2x}\right)}{x} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x \to \infty} \frac{\frac{1}{1 + e^{2x}} \cdot e^{2x} \cdot 2}{1} = \lim_{x \to \infty} \frac{2e^{2x}}{1 + e^{2x}}$$

$$= \lim_{x \to \infty} \frac{4\cancel{e^{2x}}}{2\cancel{e^{2x}}} = 2 \Rightarrow \lim_{x \to \infty} \left(1 + e^{2x}\right)^{\frac{1}{x}} = e^2$$

(d) $\lim_{x\to\infty} \dfrac{\ln(x+e^{3x})}{2x}$ $\left(\frac{\infty}{\infty}\right)$

$$= \lim_{x\to\infty} \frac{\frac{1}{x+e^{3x}} \cdot 1+3e^{3x}}{2} = \frac{1}{2} \lim_{x\to\infty} \frac{1+3e^{3x}}{x+e^{3x}} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \frac{1}{2} \lim_{x\to\infty} \frac{9e^{3x}}{1+3e^{3x}} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \frac{1}{2} \lim_{x\to\infty} \frac{27\cancel{e^{3x}}}{9\cancel{e^{3x}}} = \frac{1}{2}(3) = \frac{3}{2}$$

(e) $\lim_{x\to 0} \dfrac{\sin(x)-x}{2x^3}$ $\left(\frac{0}{0}\right)$

$$= \lim_{x\to 0} \frac{\cos x - 1}{6x^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{-\sin x}{12x}$$

$$= \frac{-1}{12} \lim_{x\to 0} \cancel{\frac{\sin x}{x}}^{\,1} \quad \text{OR} \quad \frac{-1}{12} \lim_{x\to 0} \cos x$$

$$= \frac{-1}{12} \qquad\qquad = \frac{-1}{12}$$

(f) $\lim_{x\to\infty} \frac{x^2}{x-1} - \frac{x^2}{x+5}$ $\quad (\infty - \infty)$

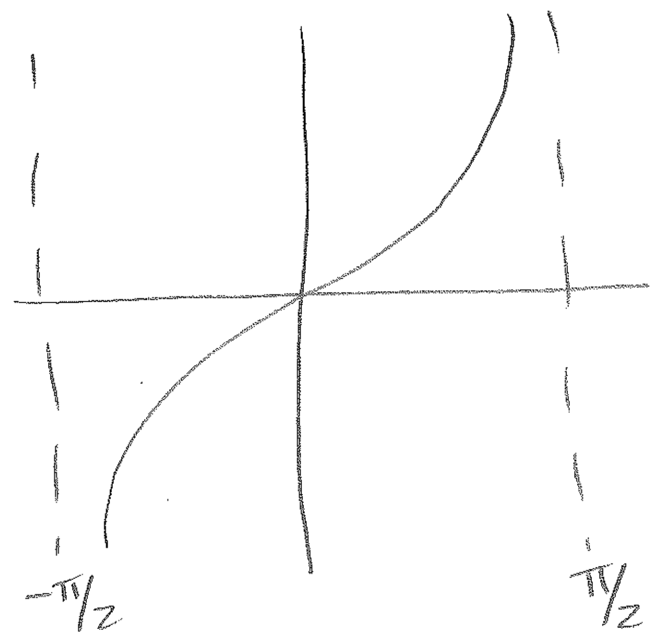
$$= \lim_{x\to\infty} \frac{x^2(x+5) - x^2(x-1)}{(x-1)(x+5)}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^3} + 5x^2 - \cancel{x^3} + x^2}{x^2+4x-5} = \lim_{x\to\infty} \frac{6x^2}{x^2+4x-5} \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{12x}{2x+4} \left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty} \frac{12}{2} = 6$$

(g) $\lim_{x\to\frac{\pi}{2}^-} (2x-\pi)\tan(x)$

$$\lim_{x\to\pi/2^-} \frac{(2x-\pi)\sin x}{\cos x} \left(\frac{0}{0}\right)$$



$$= \lim_{x\to\pi/2^-} \frac{2\sin x + (2x-\pi)\cos x}{-\sin x}$$

$$= \frac{2+0}{-1} = -2$$