## Quiz 1

1. [3pts] Given the points $P_1(1,3,2)$ and $P_2(2,3,1)$:

a). Write the vector $\overrightarrow{P_1P_2}$ in component form.

b). Find $|\overrightarrow{P_1P_2}|$.

c). Find the direction of $\overrightarrow{P_1P_2}$.

2. [2pts] Given the vectors $\vec{u} = 2\vec{i} + \vec{j}$ and $\vec{v} = -\vec{i} - 3\vec{j} + 2\vec{k}$, find cosine of the angle between $\vec{u}$ and $\vec{v}$.

3. [3pts] Find a vector orthogonal to the plane determined by the points $P(1,3,2)$, $Q(-1,0,-2)$ and $R(0,3,1)$.

4. [2pts] Find the volume of the parallelepiped determined by the vectors $\vec{u} = -3\vec{i} + \vec{j} - 2\vec{k}$, $\vec{v} = -\vec{i} + \vec{j}$, and $\vec{w} = 3\vec{i} + \vec{k}$.

## Solutions / Grading Scheme

(1) (a) $\overrightarrow{P_1P_2} = \boxed{\langle 1,0,-1\rangle}$ or $\boxed{\vec{i}-\vec{k}}$ (1 pt.)

(b) $|\overrightarrow{P_1P_2}| = \boxed{\sqrt{2}}$ (1 pt.)

(c) $\dfrac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \boxed{\left\langle \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \right\rangle}$ or $\boxed{\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{k}}$ (1 pt)

(2) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-2-3+0}{\sqrt{5}\,\sqrt{14}} = \boxed{\dfrac{-5}{\sqrt{70}}}$

(1 pt) – formula used correctly
(1 pt) – computation / final answer

(3) $\overrightarrow{PQ} = \langle -2,-3,-4\rangle$ ; $\overrightarrow{PR} = \langle -1,0,-1\rangle$ (1 pt) – forming 2 vectors in the plane

$$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -3 & -4 \\ -1 & 0 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} -3 & -4 \\ 0 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} -2 & -4 \\ -1 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix}$$

$$= \boxed{3\vec{i} + 2\vec{j} - 3\vec{k}} \text{ or } \boxed{\langle 3,2,-3\rangle}$$

(1/2 pt.) – taking cross product
(1/2 pt.) – setting up determinant
(1/3 pt.) – $\vec{i}$ component (3)
(1/3 pt.) – $\vec{j}$ component (2)
(1/3 pt.) – $\vec{k}$ component (−3)

(4)
$$\text{Vol} = \begin{vmatrix} -3 & 1 & -2 \\ -1 & 1 & 0 \\ 3 & 0 & 1 \end{vmatrix} = -3\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix} = -3(1) - (-1) - 2(-3) = \boxed{4}$$

(1 pt.) – setting up determinant
(1 pt.) – computation