NAME: Solutions

SECTION: ____

Math 2401 (D1-D3)
9/3/2014

Quiz 2

Show your work!

2pts. 1. Find parametric equations for the line going through the point $(-8,-6,-3)$ and perpendicular to the plane $9x+5y+5z=16$.

2pts. 2. a). Find a (simplified) component equation for the plane that contains the point $P_0(1,-3,-6)$ and is orthogonal to the line given by:

$$L_1:\ x=1+t;\ y=-3-4t;\ z=4t.$$

b). Is the plane in part a). perpendicular to the line:

$$L_2:\ x=2+4t;\ y=1;\ z=-6-t?$$

Justify your answer.

3pts. 3. Given the curve:

$$\vec{r}(t)=\left(\ln\left(\frac{t^2+1}{8}\right)\right)\vec{\imath}-(\cos(t))\vec{\jmath}+\left(\cos^2(t^3)\right)\vec{k},$$

find $d\vec{r}/dt$.

3pts. 4. Find:

$$\int_0^2\left[\left(2te^{5t^2}\right)\vec{\imath}+\left(e^{-t}\right)\vec{\jmath}+9\vec{k}\right]dt.$$

Solutions & Grading Scheme:

① $P(-8,-6,-3)$; $\vec{v}=\langle 9,5,5\rangle$

$x=-8+9t,\ y=-6+5t,\ z=-3+5t$

2/3 pt. - each x, y, z equation

② a). Since the plane is orthogonal to the line L_1, and $\vec{n}=\langle 1,-4,4\rangle$ is a vector parallel to the line, $\vec{n}$ is orthogonal to the plane (normal vector). So:

$1(x-1)-4(y+3)+4(z+6)=0$

$x-1-4y-12+4z+24=0$

$\boxed{x-4y+4z=-11}$

3/2 pt.
- 1 pt. - component eqn. of plane
- 1/2 pt. - simplified component eqn.

b). The plane is perpendicular to the line L_2 if L_2 is normal to the plane; since $\vec{v}=\langle 4,0,-1\rangle$ is a vector parallel to L_2, we see that L_2 is perpendicular to the plane iff $\vec{v}$ is normal to the plane. But we already know $\vec{n}=\langle 1,-4,4\rangle$ is a normal vector. Since $\vec{v}$ is clearly not a scalar multiple of $\vec{n}$, $\vec{v}$ is not parallel to $\vec{n}$, therefore L_2 is not perpendicular to the plane.

1/2 pt.

Quiz 2, continued:

(3) $\vec{r}(t) = \left(\ln\left(\frac{t^2+1}{8}\right)\right)\vec{i} - (\cos(t))\vec{j} + (\cos^2(t^3))\vec{k}$

$$\frac{d\vec{r}}{dt} = \left(\frac{1}{\frac{t^2+1}{8}} \cdot \frac{2t}{8}\right)\vec{i} - (-\sin(t))\vec{j} + \left(2\cos(t^3)(-\sin(t^3)) \cdot 3t^2\right)\vec{k}$$

$$= \boxed{\left(\frac{2t}{t^2+1}\right)\vec{i} + (\sin(t))\vec{j} - \left(6t^2\sin(t^3)\cos(t^3)\right)\vec{k}}$$

1pt. 1pt. 1pt.

(4) $\int_0^2 \left[(2te^{5t^2})\vec{i} + (e^{-t})\vec{j} + 9\vec{k}\right]dt$

$$= \boxed{\frac{1}{5}(e^{20}-1)\vec{i} + (1-e^{-2})\vec{j} + 18\vec{k}}$$

1pt. 1pt. 1pt.

$$\int_0^2 (2te^{5t^2})dt = \int_0^4 e^{5u}du = \frac{1}{5}e^{5u}\Big|_0^4 = \frac{1}{5}(e^{20}-1)$$

$t^2 = u$ | $t=0 \Rightarrow u=0$

$2t\,dt = du$ | $t=2 \Rightarrow u=4$

$$\int_0^2 e^{-t}dt = -e^{-t}\Big|_0^2 = -e^{-2} - (-1) = 1-e^{-2}$$

$$\int_0^2 9\,dt = 9t\Big|_0^2 = 18$$