NAME: Solutions

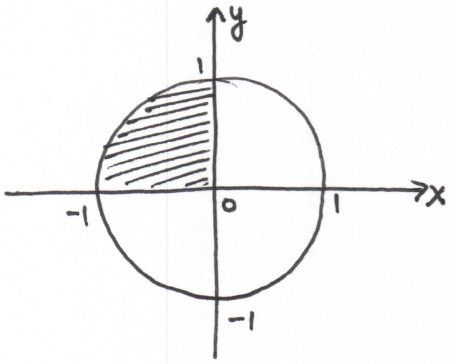
SECTION: _____

Math 2401 (D1-D3)
10/22/2014

Quiz 7

(6 pts.) 1. Compute:

$$\int_{-1}^{0}\int_{0}^{\sqrt{1-x^2}} 2\sin(x^2+y^2)\,dy\,dx.$$



$0 \le y \le \sqrt{1-x^2}$

$-1 \le x \le 0$

Polar Coordinates:

$$\int_{\pi/2}^{\pi}\int_{0}^{1} 2\sin(r^2)\cdot r\,dr\,d\theta$$

(4 pts.) 1 pt. θ bounds; 1 pt. r bounds; 1 pt. $2\sin(r^2)$; 1 pt. $r\,dr\,d\theta$

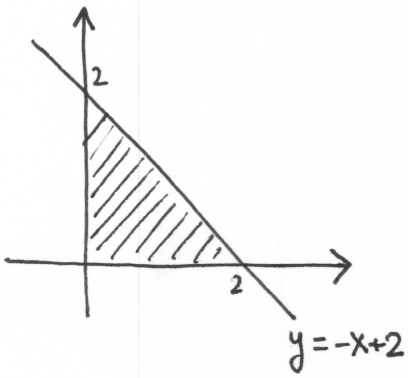
$$= \int_{\pi/2}^{\pi}\left(-\cos(r^2)\Big|_{r=0}^{r=1}\right)d\theta = \int_{\pi/2}^{\pi}\left(-\cos(1)+1\right)d\theta$$

$$= (1-\cos(1))\,\theta\Big|_{\theta=\pi/2}^{\theta=\pi}$$

$$= \boxed{\frac{\pi}{2}\left(1-\cos(1)\right)}$$

(2 pts.)

(4 pts.) 2. Let R be the region in the plane bounded by the x-axis, the y-axis, and the line $y = -x + 2$. Change the Cartesian integral $\int\int_R dx\,dy$ into an equivalent polar integral.



$$\iint_R dx\,dy = \boxed{\int_0^{\pi/2}\int_0^{\frac{2}{\sin\theta+\cos\theta}} r\,dr\,d\theta}$$

(2 pts.) – θ bounds

(2 pts.) – r bounds

$y = -x+2$

$r\sin\theta = -r\cos\theta + 2$

$r(\sin\theta+\cos\theta) = 2$

$$r = \frac{2}{\sin\theta+\cos\theta}$$