NAME: Solutions
SECTION: ____

Math 2401 (D1-D3)
11/12/2014

Quiz 9
Last quiz!!!

(10 pts.) 1. Find the *flow* (not *flux*, FLOW) of the field $\vec{F} = (x-z)\vec{i} + x\vec{k}$ over the curve C:

$$\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{k},\ 0 \le t \le 2\pi.$$

$x = \cos t \quad dx = -\sin t\, dt$
$y = 0 \quad dy = 0$
$z = \sin t \quad dz = \cos t\, dt$
$M = x - z = \cos t - \sin t$
$N = 0$
$P = x = \cos t$

$$\text{Flow} = \int_C M\,dx + N\,dy + P\,dz$$

(6pts.) setup $= \int_0^{2\pi} [(\cos t - \sin t)(-\sin t) + (\cos t)(\cos t)]\,dt$

$$= \int_0^{2\pi} (-\cos t \sin t + \sin^2 t + \cos^2 t)\,dt$$

(2pts.) $= \int_0^{2\pi} \left(-\frac{1}{2}\sin(2t) + 1\right) dt$

(2pts.) $= \left(\frac{1}{4}\cos(2t) + t\right)\Big|_0^{2\pi} = \boxed{2\pi}$

(10pts.) 2. The following field:

$$\vec{F}(x,y) = \left(2xe^{xy} + x^2ye^{xy}\right)\vec{i} + \left(x^3e^{xy} + 2y\right)\vec{j}$$

is conservative.

a). Find a potential function f for $\vec{F}$. [Hint: it might be easier if you start with $^{\partial f}/_{\partial y}$.]

$\frac{\partial f}{\partial y} = x^3e^{xy} + 2y \Rightarrow f = x^3 \cdot \frac{1}{x}e^{xy} + y^2 + C \Rightarrow f = x^2e^{xy} + y^2 + g(x)$

$\Rightarrow \frac{\partial f}{\partial x} = 2xe^{xy} + x^2ye^{xy} + g'(x)$
$\frac{\partial f}{\partial x} = 2xe^{xy} + x^2ye^{xy}$ $\Rightarrow$

$\Rightarrow g'(x) = 0 \Rightarrow g(x) = C$

(6pts.) $\boxed{f(x,y) = x^2e^{xy} + y^2 + C}$

b). Use part a). to find:

(4pts.)

$$\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r}.$$

$= f(1,1) - f(0,0) = 1e^1 + 1 - 0 = \boxed{e+1}$