

Example: Given $\vec{r}(t) = (9t+2)\vec{i} + (7-t^9)\vec{j}$, find K and \vec{N} .

$$\vec{v}(t) = \langle 9, -9t^8 \rangle$$

$$|\vec{v}(t)| = \sqrt{9^2 + 9^2 t^{16}} = 9\sqrt{1+t^{16}}$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \left\langle \frac{1}{\sqrt{1+t^{16}}}, -\frac{t^8}{\sqrt{1+t^{16}}} \right\rangle$$

$$\frac{d\vec{T}}{dt} = \left\langle -8t^{15}(1+t^{16})^{-3/2}, -8t^7(1+t^{16})^{-3/2} \right\rangle$$

$$\begin{aligned} \frac{d}{dt} \frac{1}{\sqrt{1+t^{16}}} &= \frac{d}{dt} (1+t^{16})^{-1/2} = -\frac{1}{2} (1+t^{16})^{-3/2} \cdot 16t^{15} = -8t^{15} (1+t^{16})^{-3/2} \\ \frac{d}{dt} \frac{t^8}{\sqrt{1+t^{16}}} &= \frac{8t^7 \sqrt{1+t^{16}} - t^8 \cdot \frac{16t^{15}}{2\sqrt{1+t^{16}}}}{1+t^{16}} = \frac{8t^7(1+t^{16}) - 8t^8 \cdot t^{15}}{(1+t^{16})\sqrt{1+t^{16}}} \\ &= \frac{8t^7 + 8t^{23} - 8t^{23}}{(1+t^{16})\sqrt{1+t^{16}}} = 8t^7(1+t^{16})^{-3/2} \end{aligned}$$

$$\left| \frac{d\vec{T}}{dt} \right| = \sqrt{8^2 t^{30} (1+t^{16})^{-3} + 8^2 t^{14} (1+t^{16})^{-3}}$$

$$= \sqrt{8^2 t^{14} (1+t^{16})^{-3} (t^{16} + 1)} = \sqrt{8^2 t^{14} (1+t^{16})^{-2}} = 8t^7(1+t^{16})^{-1}$$

$$\Rightarrow K = \frac{|d\vec{T}/dt|}{|\vec{v}|} = \frac{8t^7(1+t^{16})^{-1}}{9\sqrt{1+t^{16}}} = \frac{8}{9} t^7 (1+t^{16})^{-3/2}$$

$$\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|} = \frac{1}{8t^7(1+t^{16})^{-1}} \left\langle -8t^{15}(1+t^{16})^{-3/2}, -8t^7(1+t^{16})^{-3/2} \right\rangle$$

$$= \left\langle -t^8(1+t^{16})^{-1/2}, -(1+t^{16})^{-1/2} \right\rangle$$