Name: Solutions

October 29th, 2014.
Math 2401; Sections D1, D2, D3.
Georgia Institute of Technology
Exam 3

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged: ______________________

Problem	Possible Score	Earned Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Remember that you must SHOW YOUR WORK to receive credit!

Good luck!

1. [20 points] Find the volume of the region in space bounded above by the surface $z = 4\cos(x)\sin(y)$ and below by the rectangle: $0 \le x \le {}^{\pi}/_{6}$, $0 \le y \le {}^{\pi}/_{4}$.

(2pts.) $V = \iint_R f(x,y)\,dA$

(6pts.) $= \int_0^{\pi/6}\int_0^{\pi/4} 4\cos(x)\sin(y)\,dy\,dx$

(2pts.) $= \int_0^{\pi/6} -4\cos(x)\cos(y)\Big|_{y=0}^{y=\pi/4}\,dx$

(2pts.) $= \int_0^{\pi/6}\left(-4\cos(x)\cdot\frac{\sqrt{2}}{2}+4\cos(x)\right)dx$

(2pts.) $= \int_0^{\pi/6}(4-2\sqrt{2})\cos(x)\,dx$

(2pts.) $= (4-2\sqrt{2})\sin(x)\Big|_{x=0}^{x=\pi/6}$

(2pts.) $= (4-2\sqrt{2})\cdot\frac{1}{2}$

(2pts.) $= \boxed{2-\sqrt{2}}$

$\left(\text{or } \int_0^{\pi/4}\int_0^{\pi/6} 4\cos(x)\sin(y)\,dx\,dy\right)$

$\left(= \int_0^{\pi/4} 4\sin(x)\sin(y)\Big|_{x=0}^{x=\pi/6}\,dy\right)$

$\left(= \int_0^{\pi/4} 4\cdot\frac{1}{2}\sin(y)\,dy\right)$

$\left(= \int_0^{\pi/4} 2\sin(y)\,dy\right)$

$\left(= -2\cos(y)\Big|_{y=0}^{y=\pi/4}\right)$

$\left(= -2\cdot\frac{\sqrt{2}}{2}+2\cdot 1\right)$

$\left(= \boxed{2-\sqrt{2}}\right)$

2 pts. Correct bounds for x

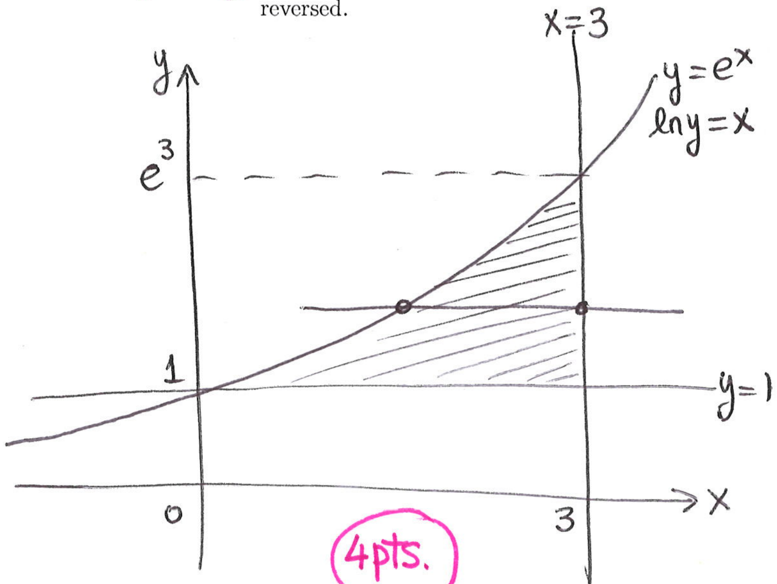
2pts. correct bounds for y

2pts. correct order of dx, dy.

2. [20 points] Given the integral below:

$$\int_0^3 \int_1^{e^x} \frac{1}{y}\, dy\, dx.$$

[12 pts.] a). Sketch the region of integration and write an equivalent double integral with the order of integration reversed.



4pts.

$1 \le y \le e^x$

$0 \le x \le 3$

Horizontal Cross-Sections:

$$\boxed{\int_1^{e^3} \int_{\ln(y)}^{3} \frac{1}{y}\, dx\, dy}$$

8pts. — 4pts. bounds for x; 4pts. bounds for y

[8 pts.] b). Use either one of the two versions of the integral to compute its value.

$$\int_0^3 \int_1^{e^x} \frac{1}{y}\, dy\, dx$$

$$= \int_0^3 \ln(y) \Big|_{y=1}^{y=e^x} dx \quad \text{2pts.}$$

$$= \int_0^3 \left(\ln(e^x) - \ln(1)\right) dx \quad \text{2pts.}$$

$$= \int_0^3 (x)\, dx \quad \text{2pts.}$$

$$= \frac{x^2}{2} \Big|_0^3 \quad \text{1pt.}$$

$$= \boxed{9/2} \quad \text{1pt.}$$

or

$$\int_1^{e^3} \int_{\ln(y)}^{3} \frac{1}{y}\, dx\, dy$$

$$= \int_1^{e^3} \frac{1}{y} x \Big|_{x=\ln(y)}^{x=3} dy \quad \text{2pts.}$$

$$= \int_1^{e^3} \left(\frac{3}{y} - \frac{1}{y}\ln(y)\right) dy \quad \text{2pts.}$$

$$= \left(3\ln(y) - \frac{\ln^2(y)}{2}\right) \Big|_{y=1}^{y=e^3} \quad \text{2pts.}$$

$$= 3\ln(e^3) - \frac{(\ln(e^3))^2}{2} - 3\ln(1) + \frac{(\ln(1))^2}{2} \quad \text{1pt.}$$

$$= 3 \cdot 3 - \frac{3^2}{2}$$

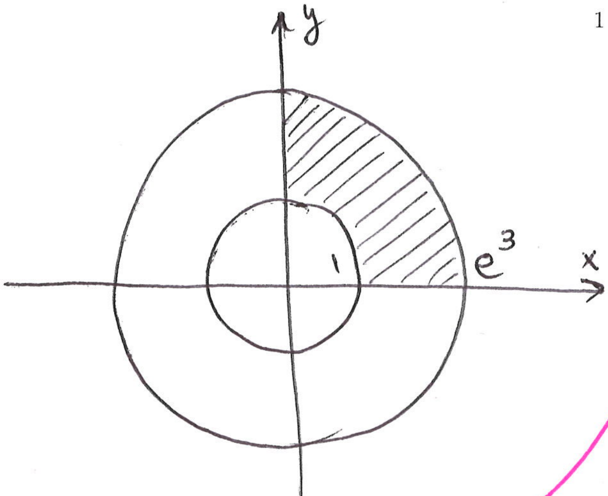
$$= \boxed{9/2} \quad \text{1pt.}$$

3. [20 points] Compute:

$$\iint_R \frac{\ln(x^2+y^2)}{x^2+y^2}\, dA,$$

where R is the region in the xy-plane given by:

$$1 \le x^2+y^2 \le e^6;\ x > 0;\ y > 0.$$



$$\iint_R \frac{\ln(x^2+y^2)}{x^2+y^2}\, dA$$

10 pts. $= \int_0^{\pi/2} \int_1^{e^3} r \cdot \frac{\ln(r^2)}{r^2}\, dr\, d\theta$

3 pts. $= \int_0^{\pi/2} \int_1^{e^3} \frac{2\ln(r)}{r}\, dr\, d\theta$

3 pts. $= \int_0^{\pi/2} \ln^2(r) \Big|_{r=1}^{r=e^3}\, d\theta$

2 pts. $= \int_0^{\pi/2} (9)\, d\theta$

2 pts. $= 9\theta \Big|_{\theta=0}^{\theta=\pi/2} = \boxed{\frac{9\pi}{2}}$

bounds for r 3 pts.

bounds for θ 3 pts.

$\frac{\ln(r^2)}{r^2}$ 2 pts.

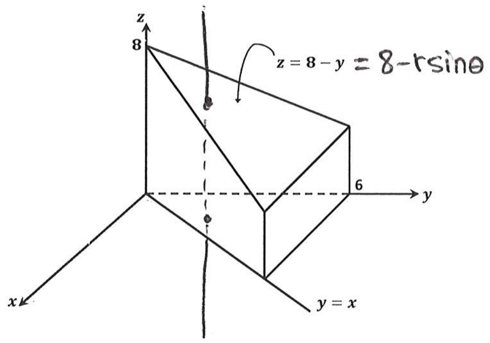
$\underline{r\,dr\,d\theta}$ 2 pts.

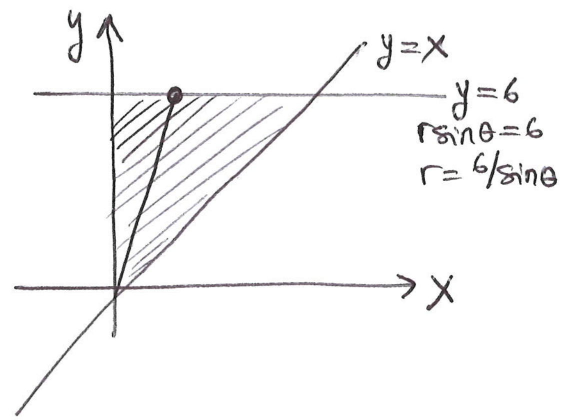
[12pts.]

4. [20 points] a). Set up the integral for evaluating

$$\iiint_D r\,dz\,dr\,d\theta$$

over the solid D shown in the figure below: D is the prism whose base is the triangle in the xy-plane bounded by the y-axis and the lines $y = x$, $y = 6$, and whose top lies in the plane $z = 8 - y$.





$$\boxed{\int_{\pi/4}^{\pi/2}\int_0^{\frac{6}{\sin\theta}}\int_0^{8-r\sin\theta} r\,dz\,dr\,d\theta}$$

4pts. – bounds for z

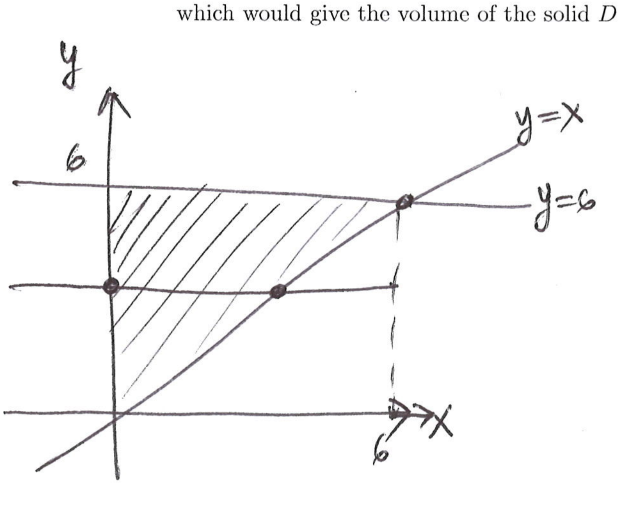
4pts. – bounds for r

4pts. – bounds for θ

[8pts.] b). Set up a double integral of the form:

$$\iint_R f(x,y)\,dx\,dy$$

which would give the volume of the solid D in part a).



$$\boxed{\underbrace{\int_0^6}_{3pts.}\underbrace{\int_0^y}_{3pts.}\underbrace{(8-y)}_{2pts.}\,dx\,dy}$$

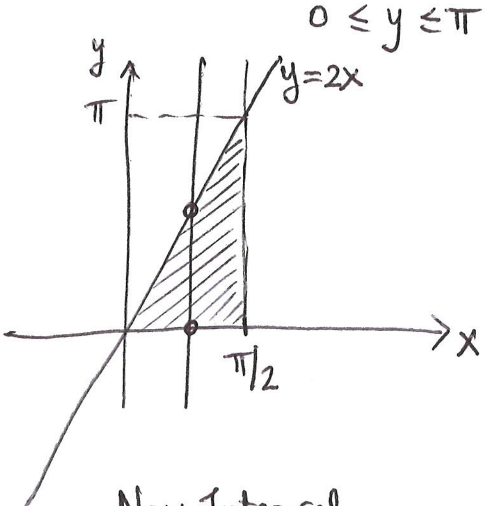
5. [20 points] Compute the integral:

$$\int_0^1 \int_0^\pi \int_{y/2}^{\pi/2} \frac{z\cos(x)}{x}\, dx\, dy\, dz.$$

2pts. Switch order of integration between x and y, since we cannot integrate $\frac{\cos(x)}{x}$ directly.

Region: $y/2 \le x \le \pi/2$

$0 \le y \le \pi$



Vertical Cross-Sections:

$$\int_0^{\pi/2}\int_0^{2x} dy\, dx$$

3pts.

3pts.

New Integral:

2pts. $$\int_0^1\int_0^{\pi/2}\int_0^{2x} \frac{z\cos(x)}{x}\, dy\, dx\, dz = \int_0^1\int_0^{\pi/2} \frac{z\cos(x)}{x}\, y \Big|_{y=0}^{y=2x} dx\, dz$$ 2pts.

2pts. $$= \int_0^1\int_0^{\pi/2} \frac{z\cos(x)}{x}\cdot 2x\, dx\, dz = \int_0^1\int_0^{\pi/2} 2z\cos(x)\, dx\, dz$$

2pts. $$= \int_0^1 2z\sin(x)\Big|_{x=0}^{x=\pi/2} dz$$

2pts. $$= \int_0^1 2z\left(\sin \pi/2 - \sin 0\right) dz$$

$$= \int_0^1 2z\, dz$$

2pts. $$= z^2\Big|_0^1 = \boxed{1}$$