Name: ______________________________

December 9$^{\text{th}}$, 2014.
Math 2401; Sections D1, D2, D3.
Georgia Institute of Technology
FINAL EXAM

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged: ______________________________

Problem	Possible Score	Earned Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
Total	140	

Remember that you must SHOW YOUR WORK to receive credit!

Good luck!

1. [10 points] Consider the lines:

$$L_1: \ x = 1 + t; \ \ y = 2 - t; \ \ z = 3t$$

$$L_2: \ x = 2 - s; \ \ y = 1 + 2s; \ \ z = 3 + s.$$

(a). Find the point of intersection of these lines.
(b). Find an equation for the plane determined by these lines.

2. [10 points] Consider the curve:

$$\vec{r}(t) = (2\sin t)\vec{i} + (t^4 - 4\cos t)\vec{j} + (e^{2t})\vec{k}.$$

(a). Find the velocity vector $\vec{v}(t)$ for this curve.
(b). Find the acceleration vector $\vec{a}(t)$ for this curve.
(c). Find the angle θ between $\vec{v}(0)$ and $\vec{a}(0)$.

3. [10 points] Find the length of the curve:

$$\vec{r}(t) = \left\langle \cos^3 t, \sin^3 t \right\rangle, \ \ \frac{\pi}{2} \leq t \leq \pi.$$

4. [10 points] Find the directions $\vec{u}$ such that the directional derivative:

$$D_{\vec{u}}f(1,1) = -1,$$

where $f(x,y) = x^2y - 2y + xy^2$.

5. [10 points] Find the points where extrema occur for the function:

$$f(x, y) = x^3 y^5,$$

subject to the constraint:

$$x + y = 8.$$

6. [10 points] Find the minimum and maximum of the function:

$$f(x, y) = (x - 3)^2 + (y - 1)^2$$

on the triangular plate in the first Quadrant determined by the lines $y = 0$, $x = 4$, and $y = x$.

7. [10 points] Sketch the region of integration and reverse the order of integration:

$$\int_0^9 \int_{y/3}^{\sqrt{y}} dx\, dy.$$

Do not evaluate the integrals.

8. [10 points] Evaluate:

$$\int_1^{e^{10}} \int_1^{e^4} \int_1^{e^3} \frac{1}{xyz}\, dx\, dy\, dz.$$

9. [10 points] The following field is conservative:

$$\vec{F}(x,y,z) = (2xy + \cos(x))\,\vec{i} + \left(x^2\right)\vec{j} + \left(-\sin(z)e^{\cos(z)}\right)\vec{k}.$$

(a). Find a potential function f for this field.

(b). Evaluate:

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is a smooth curve from $(\pi, 0, 0)$ to $(\pi, 0, \pi)$.

10. [10 points] Find the outward flux of the field:

$$\vec{F}(x, y, z) = \sqrt{x^2 + y^2 + z^2}(x\vec{i} + y\vec{j} + z\vec{k})$$

over the boundary of the region D in space, given by:

$$D : 4 \leq x^2 + y^2 + z^2 \leq 5.$$

11. [10 points] Find the circulation of the field:

$$\vec{F}(x,y,z) = x^2\vec{i} + 3x\vec{j} + z^2\vec{k}$$

around the ellipse:

$$C : 16x^2 + y^2 = 3$$

in the xy-plane, oriented counterclockwise when viewed from above (from the positive side of the z-axis). You may use the fact that the area of an ellipse with axes a and b is πab.

12. [10 points] Find:

$$\oint_C x^2y^3\,dx + (x^3y^2 + x)\,dy,$$

where C is the counterclockwise oriented boundary of a square of side length 3 in the xy-plane.

13. [10 points] Find the limit, if it exists:

$$\lim_{(x,y)\to(1,1)} \frac{x^2y - xy^2}{\sqrt{x} - \sqrt{y}}.$$

14. [10 points] Find:

$$\int_0^\infty e^{-3x^2}\, dx.$$