

Worksheet 1 - Review

1. Find all the values of  $x$  where the tangent lines to  $y = \frac{x^3}{3} + 5x$  and  $y = 3x^2$  are parallel.

Recall: - Slope of tangent line to a graph is given by the derivative  
- Two lines are parallel when their slopes are equal

$$y = \frac{x^3}{3} + 5x \Rightarrow y' = x^2 + 5$$

$$y = 3x^2 \Rightarrow y' = 6x$$

$$x^2 + 5 = 6x$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0 \Rightarrow \boxed{x = 1, 5}$$

2. Given that  $f'(x) = 2e^x - 3$ , and that  $f(0) = 7$ , find  $f(x)$ .

$$f(x) = \int (2e^x - 3) dx = 2e^x - 3x + C$$

$$\left. \begin{array}{l} f(0) = 2e^0 - 3 \cdot 0 + C = 2 + C \\ f(0) = 7 \end{array} \right\} \Rightarrow C = 5 \Rightarrow \boxed{f(x) = 2e^x - 3x + 5}$$

3. Find  $\frac{dy}{dx}$  for  $y = x^{3x}$ .

$$y = x^{3x} = (e^{\ln(x)})^{3x} = e^{\ln(x) \cdot 3x}$$

$$y' = e^{\ln(x) \cdot 3x} [\ln(x) \cdot 3x]' = x^{3x} \left[ \frac{1}{x} \cdot 3x + \ln(x) \cdot 3 \right]$$

$$= \boxed{x^{3x} [3 + 3 \ln(x)]}$$

4. For what value of  $c$  is the function below continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} cx + 4 & , \text{ if } x < 1 \\ x^2 + 2c & , \text{ if } x \geq 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (cx + 4) = c + 4$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 + 2c) = 1 + 2c = f(1)$$

$$c + 4 = 1 + 2c \Rightarrow \boxed{c = 3}$$

Recall: A function is continuous at a point  $a$  in the domain if and only if

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

5. Find the derivative of  $f(x) = \arctan(\sin(x))$ .

$$f'(x) = \frac{\cos(x)}{1 + \sin^2(x)} \quad \text{Chain Rule}$$

6. Find the derivative of  $f(x) = 3^{x^2 - 3x + 4}$ .

$$f'(x) = 3^{x^2 - 3x + 4} \cdot \ln(3) \cdot (2x - 3) \quad \text{Chain Rule}$$

7. Find the integral:

$$\int_1^{e^4} \frac{1}{2x\sqrt{\ln(x)}} dx = \int_0^4 \frac{1}{2\sqrt{u}} du = \sqrt{u} \Big|_0^4 = \boxed{2}$$

$$u = \ln(x)$$

$$du = \frac{1}{x} dx$$

Substitution

$$x = 1 \Rightarrow u = \ln(1) = 0$$

$$x = e^4 \Rightarrow u = \ln(e^4) = 4$$

8. Find the integral:

$$\int \frac{x-2}{x^2-6x+10} dx = \int \frac{x-2}{(x-3)^2+1} dx = \int \frac{(x-3)+1}{(x-3)^2+1} dx$$

$$= \int \frac{x-3}{(x-3)^2+1} dx + \int \frac{1}{(x-3)^2+1} dx = \boxed{\frac{1}{2} \ln(x^2-6x+10) + \arctan(x-3) + C}$$

$$u = (x-3)^2 + 1$$

$$du = 2(x-3) dx$$

$$\frac{1}{2} du = (x-3) dx$$

$$\int \frac{1}{u} \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \ln|u| = \frac{1}{2} \ln|(x-3)^2+1| + C$$

$$= \frac{1}{2} \ln(x^2-6x+10) + C$$

9. Find the integral:

$$\begin{aligned}
 \int x^6 \ln(x) dx &= \int \left(\frac{x^7}{7}\right)' \ln(x) dx = \frac{x^7}{7} \ln(x) - \int \frac{x^7}{7} (\ln(x))' dx \\
 &= \frac{x^7}{7} \ln(x) - \int \frac{x^7}{7} \cdot \frac{1}{x} dx = \frac{x^7}{7} \ln(x) - \frac{1}{7} \int x^6 dx \\
 &= \frac{x^7}{7} \ln(x) - \frac{1}{7} \cdot \frac{1}{7} x^7 + C \\
 &= \boxed{\frac{x^7}{7} \ln(x) - \frac{x^7}{49} + C} \quad \text{Integration by Parts}
 \end{aligned}$$

10. Find the integral:

$$\int \pi e^{25} dx = \boxed{\pi e^{25} x + C}$$

11. Find the integral:

$$\begin{aligned}
 \int \frac{\sin^3(x)}{\cos(x)} dx &= \int \frac{\sin^2(x)}{\cos(x)} \cdot \sin(x) dx = \int \frac{1 - \cos^2(x)}{\cos(x)} \cdot \sin(x) dx \\
 &= \int \frac{1 - u^2}{u} (-du) \quad \begin{array}{l} u = \cos(x) \\ du = -\sin(x) dx \end{array} \\
 &= - \int \left(\frac{1}{u} - u\right) du \\
 &= - \left(\ln|u| - \frac{u^2}{2}\right) + C \\
 &= \boxed{-\ln|\cos(x)| + \frac{\cos^2(x)}{2} + C}
 \end{aligned}$$

12. Find the integral:

$$\begin{aligned}
 \int \arctan(x) dx &= \int (x)' \arctan(x) dx = x \arctan(x) - \int x (\arctan'(x)) dx \\
 &= x \arctan(x) - \int \frac{x}{1+x^2} dx \\
 &= x \arctan(x) - \int \frac{1}{u} \cdot \frac{1}{2} du \\
 &= x \arctan(x) - \frac{1}{2} \ln|u| + C \\
 &= \boxed{x \arctan(x) - \frac{1}{2} \ln(1+x^2) + C}
 \end{aligned}$$

$$\begin{array}{l}
 u = 1+x^2 \\
 du = 2x dx \\
 \frac{1}{2} du = x dx
 \end{array}$$

13. Find the integral:

$$\int \frac{1}{2\sqrt{x+1} + x\sqrt{x+1}} dx = \int \frac{1}{\sqrt{x+1}(x+2)} dx = \int \frac{1}{\sqrt{x+1}(1+(\sqrt{x+1})^2)} dx$$

$$\begin{aligned} u &= \sqrt{x+1} \\ du &= \frac{1}{2\sqrt{x+1}} dx \\ 2du &= \frac{1}{\sqrt{x+1}} dx \end{aligned}$$

$$= \int \frac{1}{1+u^2} 2du$$

$$= 2 \arctan(u) + C$$

$$= \boxed{2 \arctan(\sqrt{x+1}) + C}$$

14. Find the integral:

$$\int \cos(\sqrt{x}) dx$$

$$\begin{aligned} u &= \sqrt{x} \\ u^2 &= x \end{aligned}$$

$$2u du = dx$$

$$\int \cos(\sqrt{x}) dx = \int \cos(u) \cdot 2u du = 2 \int u \cos(u) du$$

$$= 2 \int u (\sin(u))' du$$

$$= 2 \left[ u \sin(u) - \int u' \sin(u) du \right]$$

$$= 2 \left[ u \sin(u) - \int \sin(u) du \right]$$

$$= 2 \left[ u \sin(u) + \cos(u) + C \right]$$

$$= \boxed{2\sqrt{x} \sin(\sqrt{x}) + 2\cos(\sqrt{x}) + C}$$