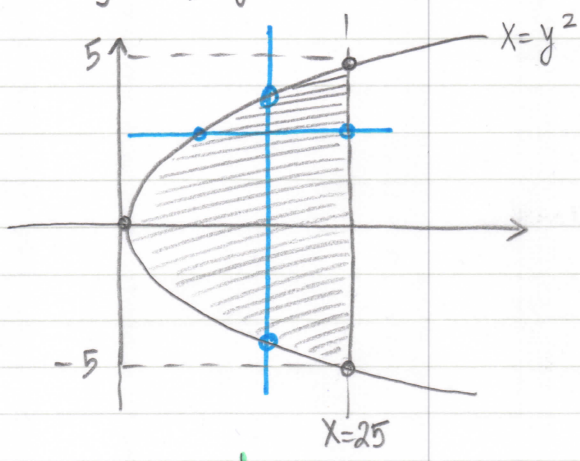
Worksheet 3 - Solutions

① $-5 \le y \le 5$, $y^2 \le x \le 25$



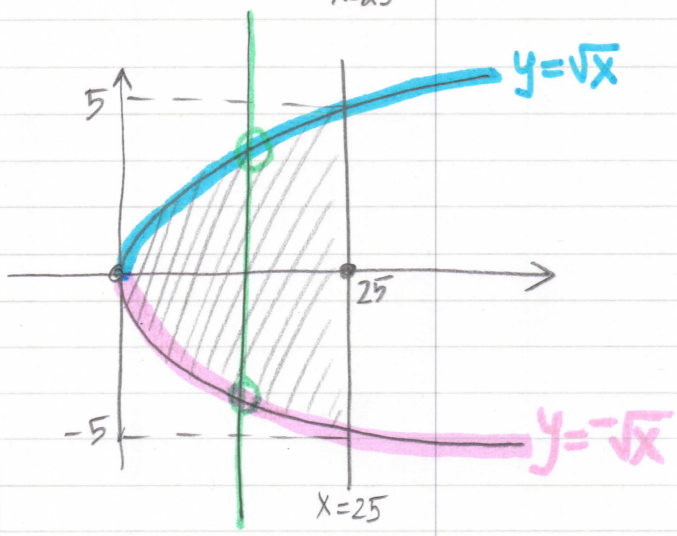
Horizontal Cross-Sections:

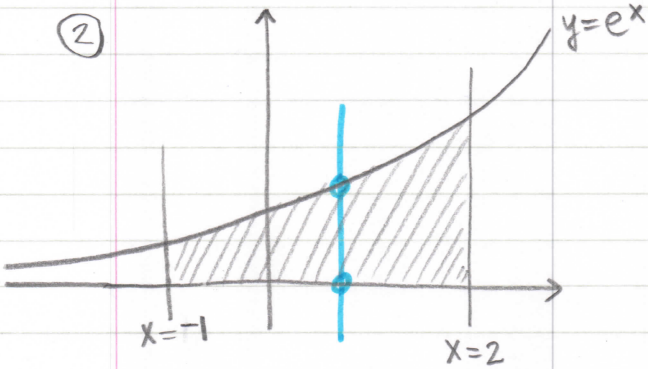
$$\int_{-5}^{5}\int_{y^2}^{25} dx\,dy$$

Vertical Cross-Sections:

Careful: $x=y^2$ has two different expressions of the inverse function, depending on y being positive or negative

$$\int_{0}^{25}\int_{-\sqrt{x}}^{\sqrt{x}} dy\,dx$$



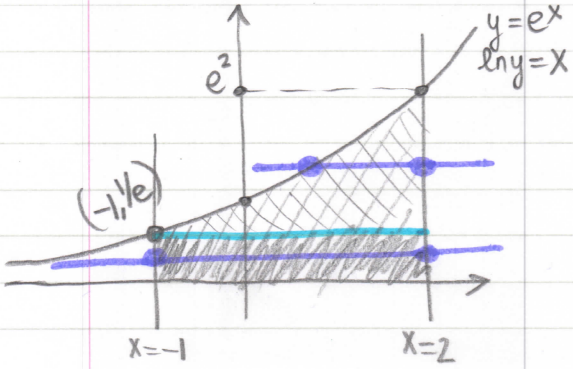
② 

Vertical Cross-Sections:

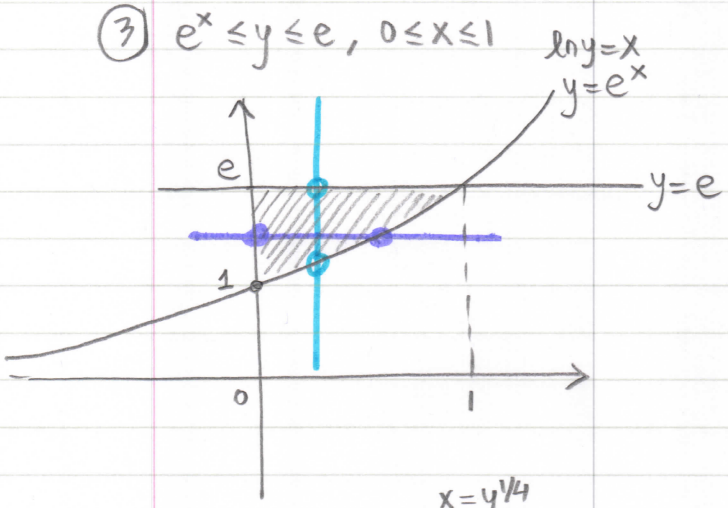
$$\int_{-1}^{2}\int_{0}^{e^x} dy\,dx$$

Horizontal Cross-Sections:

$$\int_{0}^{1/e}\int_{-1}^{2} dx\,dy + \int_{1/e}^{e^2}\int_{\ln y}^{2} dx\,dy$$



③ $e^x \le y \le e$, $0 \le x \le 1$



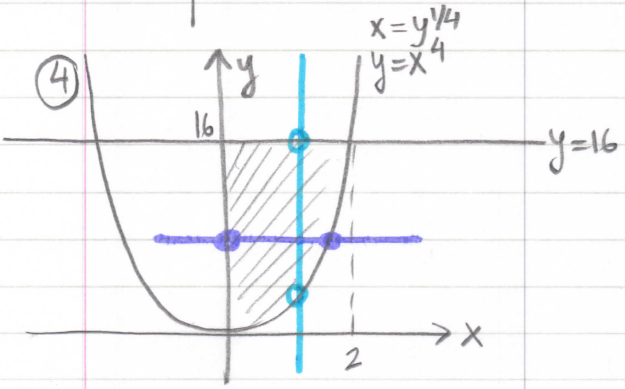
Vertical Cross-Sections:

$$\int_0^1 \int_{e^x}^{e} dy\, dx$$

Horizontal Cross-Sections:

$$\int_1^e \int_0^{\ln y} dx\, dy$$

④



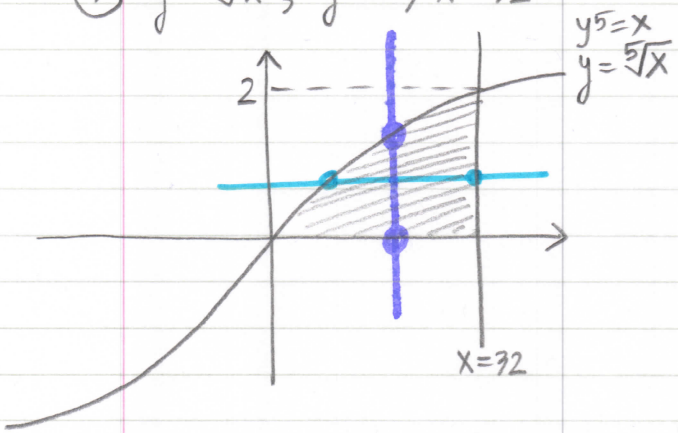
Vertical Cross-Sections:

$$\int_0^2 \int_{x^4}^{16} dy\, dx$$

Horizontal Cross-Sections:

$$\int_0^{16} \int_0^{y^{1/4}} dx\, dy$$

⑤ $y = \sqrt[5]{x}$, $y = 0$, $x = 32$



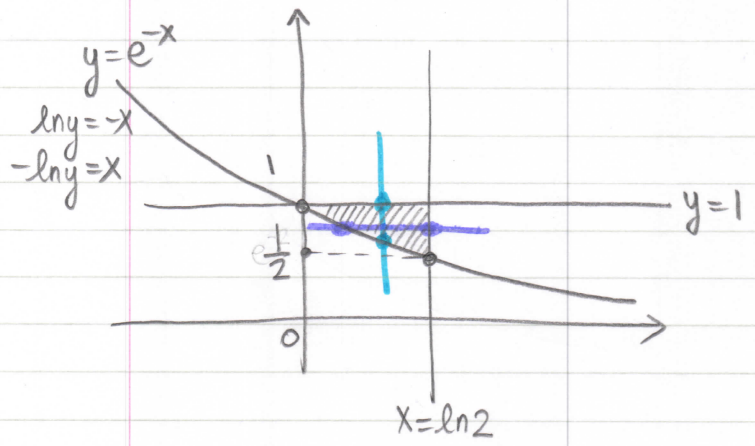
Vertical Cross-Sections:

$$\int_0^{32} \int_0^{\sqrt[5]{x}} dy\, dx$$

Horizontal Cross-Sections:

$$\int_0^2 \int_{y^5}^{32} dx\, dy$$

⑥ $y = e^{-x}$, $y = 1$, $x = \ln(2)$



Vertical Cross-Sections:

$$\int_0^{\ln 2} \int_{e^{-x}}^{1} dy\, dx$$

Horizontal Cross-Sections:

$$\int_{1/2}^{1} \int_{-\ln y}^{\ln 2} dx\, dy$$

⑦ $R: 0 \le y \le 10,\ 0 \le x \le 1$

$$\iint_R \frac{xy^4}{x^2+1}\, dA = \int_0^1 \int_0^{10} \frac{x}{x^2+1} y^4\, dy\, dx = \int_0^1 \left(\frac{x^2}{x^2+1} \frac{y^5}{5} \Big|_{y=0}^{y=10} \right) dx$$

$$= \int_0^1 \frac{x}{x^2+1} (20{,}000)\, dx$$

$$= \frac{1}{2} \ln(x^2+1) \cdot (20{,}000) \Big|_0^1$$

$$= (10{,}000) \ln(x^2+1) \Big|_0^1 = (10{,}000)(\ln 2 - \underbrace{\ln 1}_{0})$$

$$= \boxed{(10{,}000) \ln 2}$$