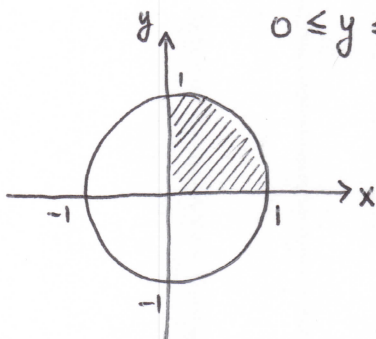


① $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2) dx dy$

$0 \leq x \leq \sqrt{1-y^2}$
 $0 \leq y \leq 1$

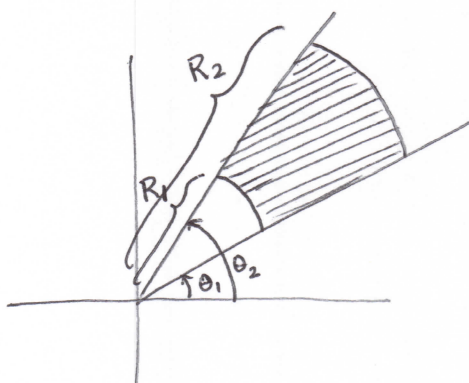


Polar Coordinates:

$$\begin{aligned} & \int_0^{\pi/2} \int_0^1 \cos(r^2) \cdot r dr d\theta \\ &= \int_0^{\pi/2} \left(\frac{1}{2} \sin(r^2) \Big|_{r=0}^{r=1} \right) d\theta \\ &= \int_0^{\pi/2} \frac{1}{2} \sin(1) d\theta \\ &= \frac{1}{2} \sin(1) \theta \Big|_{\theta=0}^{\theta=\pi/2} = \boxed{\frac{\pi}{4} \sin(1)} \end{aligned}$$

② Area of a circular section

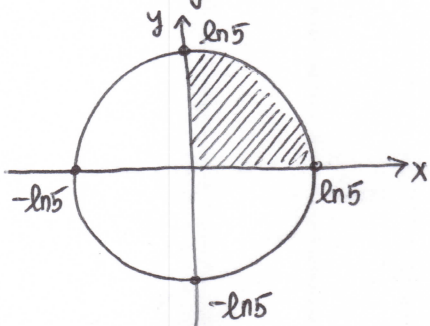
$R_1 \leq r \leq R_2 ; \theta_1 \leq \theta \leq \theta_2 :$



$$\begin{aligned} A &= \int_{\theta_1}^{\theta_2} \int_{R_1}^{R_2} r dr d\theta \\ &= \int_{\theta_1}^{\theta_2} \frac{r^2}{2} \Big|_{r=R_1}^{r=R_2} d\theta \\ &= \int_{\theta_1}^{\theta_2} \frac{R_2^2 - R_1^2}{2} d\theta \\ &= \frac{R_2^2 - R_1^2}{2} \theta \Big|_{\theta=\theta_1}^{\theta_2} = \boxed{\frac{1}{2} (R_2^2 - R_1^2) (\theta_2 - \theta_1)} \end{aligned}$$

③ $\int_0^{\ln 5} \int_0^{\sqrt{(\ln 5)^2 - y^2}} e^{\sqrt{x^2+y^2}} dx dy$

$0 \leq x \leq \sqrt{(\ln 5)^2 - y^2}$
 $0 \leq y \leq \ln 5$



Polar Coordinates:

$$\begin{aligned} & \int_0^{\pi/2} \int_0^{\ln 5} e^r \cdot r dr d\theta \\ &= \int_0^{\pi/2} \int_0^{\ln 5} (e^r)' \cdot r dr d\theta \\ &= \int_0^{\pi/2} \left(e^r \cdot r \Big|_0^{\ln 5} - \int_0^{\ln 5} e^r dr \right) d\theta \\ &= \int_0^{\pi/2} \left(5 \ln 5 - e^r \Big|_0^{\ln 5} \right) d\theta \\ &= \int_0^{\pi/2} (5 \ln 5 - 5 + 1) d\theta = \boxed{\frac{\pi}{2} (5 \ln 5 - 4)} \end{aligned}$$

④ Area of region enclosed by the curves:

$$r = \cos \theta \quad \text{and} \quad r = \sin \theta$$

What do these curves look like?

$$r = \cos \theta \quad | \cdot r$$

$$r^2 = r \cos \theta$$

$$x^2 + y^2 = x$$

$$x^2 - x + y^2 = 0$$

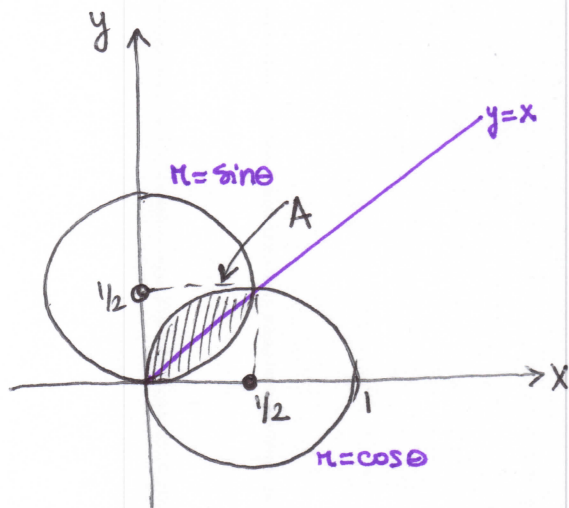
Complete the Square:

$$x^2 - 2 \cdot \frac{1}{2} x + y^2 = 0$$

$$x^2 - 2 \cdot \frac{1}{2} x + \frac{1}{4} + y^2 = \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$$

Circle centered at $(\frac{1}{2}, 0)$, w/ radius $\frac{1}{2}$.



Thus the area of the original region is

$$\boxed{\frac{\pi - 2}{8}}$$

$$r = \sin \theta \quad | \cdot r$$

$$r^2 = r \sin \theta$$

$$x^2 + y^2 = y$$

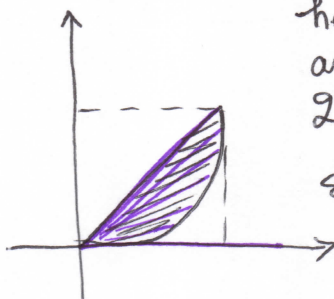
$$x^2 + y^2 - y = 0$$

$$x^2 + y^2 - 2 \cdot \frac{1}{2} y + \frac{1}{4} = \frac{1}{4}$$

$$x^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

Circle centered at $(0, \frac{1}{2})$
w/ radius $\frac{1}{2}$.

To find the area of the region,
we look at a half of the region:



here θ varies from 0 to $\pi/4$
and r "ends" on the
quarter of the circle

$r = \sin \theta$,
so the area of this
half-region is:

$$\int_0^{\pi/4} \int_0^{\sin \theta} r \, dr \, d\theta = \int_0^{\pi/4} \frac{r^2}{2} \Big|_{r=0}^{\sin \theta} d\theta$$

$$= \frac{1}{2} \int_0^{\pi/4} \sin^2 \theta \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/4} \frac{1 - \cos(2\theta)}{2} d\theta$$

$$= \frac{1}{4} \left(\theta - \frac{1}{2} \sin(2\theta) \right) \Big|_{\theta=0}^{\pi/4}$$

$$= \frac{1}{4} \left(\frac{\pi}{4} - \frac{1}{2} \sin\left(\frac{\pi}{2}\right) \right) = \frac{1}{4} \left(\frac{\pi}{4} - \frac{1}{2} \right) = \frac{\pi - 2}{16}$$

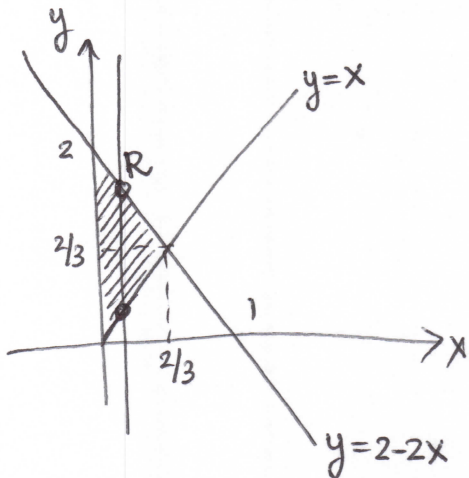
$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$(5) \int_{\pi/4}^{\pi/2} \int_0^{\frac{2}{\sin\theta + 2\cos\theta}} r^3 \cos^2\theta \, dr \, d\theta$$

Sketch the region:

$$0 \leq r \leq \frac{2}{\sin\theta + 2\cos\theta}$$



$$r = \frac{2}{\sin\theta + 2\cos\theta}$$

$$r\sin\theta + 2r\cos\theta = 2$$

$$y + 2x = 2$$

$$\boxed{y = 2 - 2x}$$

Point of intersection:

$$x = 2 - 2x$$

$$3x = 2$$

$$x = \frac{2}{3}, y = \frac{2}{3}$$

Convert integral to rectangular:

$$\int_{\pi/4}^{\pi/2} \int_0^{\frac{2}{\sin\theta + 2\cos\theta}} r^3 \cos^2\theta \, dr \, d\theta = \int_0^{2/3} \int_x^{2-2x} x^2 \, dy \, dx$$

$$\underbrace{r^2 \cos^2\theta}_{x^2} \underbrace{(r \, dr \, d\theta)}_{dy \, dx}$$

$$= \int_0^{2/3} x^2 y \Big|_{y=x}^{y=2-2x} dx = \int_0^{2/3} (x^2(2-2x) - x^3) dx$$

$$= \int_0^{2/3} (2x^2 - 3x^3) dx = \left(\frac{2}{3}x^3 - \frac{3}{4}x^4 \right) \Big|_0^{2/3} = \left(\frac{2}{3} \right)^4 - \frac{3}{4} \left(\frac{2}{3} \right)^4 = \frac{1}{4} \left(\frac{2}{3} \right)^4$$

$$= \boxed{\frac{4}{81}}$$