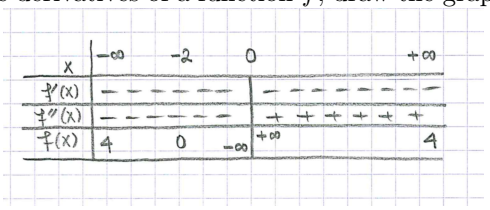
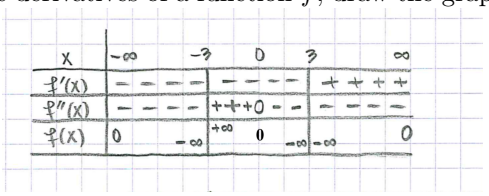
Exam 2 Review: Worksheet 3

1. Given the table below describing the derivatives of a function f, draw the graph of f and mark any asymptotes, inflection points, local minima or local maxima.



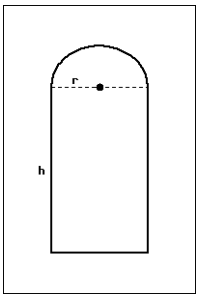
2. Given the table below describing the derivatives of a function f, draw the graph of f and mark any asymptotes, inflection points, local minima or local maxima.



3. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

4. We want to construct a box with a square base and we only have 10 m^2 of material to use in construction of the box. Assuming that all the material is used in the construction process determine the maximum volume that the box can have.

5. A Norman window is a single window that has the shape of a semicircle above a rectangle so that the diameter of the semicircle is equal to the width of the rectangle.



Suppose that a Norman window has an outside perimeter of 4 meters.

a). What is the area of the entire window as a function of r only?
b). What is the largest possible area of a Norman window with an outside perimeter of 4 meters?

6. Determine the point on the line $2x + y + 3 = 0$ closest to the point $(-2, -3)$.

7. Estimate the area under the graph of $f(x) = x^2 + 3x$ from $x = 2$ to $x = 8$ using the areas of 3 rectangles of equal width, with heights of the rectangles determined by the height of the curve at: a). The left endpoints; b). The right endpoints.

Evaluate the integrals below by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using simple formulas from geometry.

8. $\displaystyle\int_0^3 |4x - 9|\, dx$

9. $\displaystyle\int_{-1}^2 |2x - 3|\, dx$

10. $\displaystyle\int_{-8}^8 \sqrt{64 - x^2}\, dx$

11. If $g(x) = \int_1^x \frac{1}{1+t^4}\, dt$ find $g'(x)$.

Evaluate the integrals below:

12. $\displaystyle\int_0^6 (10x^2 - 6x + 2)\, dx$

13. $\displaystyle\int_{-\pi/4}^{\pi/3} (\sec^2(x) + \sin(x))\, dx$

14. $\displaystyle\int_1^{\sqrt{7}} \frac{15s^5 + 3\sqrt{s}}{s^5}$

15. $\displaystyle\int_3^1 \left(\frac{3}{x^2} - 7\right) dx$

16. Use a Riemann sum with three rectangles to *underestimate* the area under the graph of $y = x^2 + 3$ between $x = -1$ and $x = 2$.

17. Express the following sum using the sigma notation: [Multiple Choice]

$$2 + 6 + 12 + 20 + 30$$

(a). $\displaystyle\sum_{k=1}^{5}(k^2 - k)$

(b). $\displaystyle\sum_{k=0}^{4}(k^2 + k)$

(c). $\displaystyle\sum_{k=0}^{4}(2^k - k)$

(d). $\displaystyle\sum_{k=1}^{5}(k^2 + k)$