TUESDAY 10/10

Substitution

1. $\int \frac{1}{x^2}\cos\left(\frac{1}{x}\right)dx = \boxed{-\sin\left(\frac{1}{x}\right)+C}$

Trig Integral

2. $\int \cos^2 x \sin^3 x\, dx$

$u = \cos x$

$= \int \cos^2 x(1-\cos^2 x)\sin x\, dx$

$= -\int u^2(1-u^2)du = +\int (u^4-u^2)du$

$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$

$= \boxed{\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$

By Parts

3. $\int x^2 e^x dx$

$u = x^2 \quad ; du = 2x\,dx$
$dv = e^x dx \,; \ v = e^x$

$= x^2e^x - 2\int xe^x dx$

$u = x \quad ; du = dx$
$dv = e^x ; \ v = e^x$

$= x^2e^x - 2\left(xe^x - \int e^x dx\right)$

$= \boxed{x^2e^x - 2xe^x + 2e^x + C}$

Improper Integral + By Parts

4. $\int_0^\infty xe^{-x}dx = I$

$\int_0^t xe^{-x}dx = \int_0^t x(-e^{-x})'dx = -xe^{-x}\Big|_0^t + \int_0^t e^{-x}dx$

$= -te^{-t} - e^{-x}\Big|_0^t = -te^{-t} - e^{-t} + 1 = 1 - \frac{t+1}{e^t}$

$I = \lim_{t\to\infty}\left(1 - \frac{t-1}{e^t}\right) \overset{l'H}{\underset{\frac{\infty}{\infty}}{=}} 1 - \underbrace{\lim_{t\to\infty}\left(\frac{1}{e^t}\right)}_{0} = \boxed{1}.$

Trig Substitution

5. $\int \frac{1}{x}\sqrt{9x^2-4}\, dx$

$= \int \frac{3}{x}\sqrt{x^2 - \frac{4}{9}}\,dx$

$x = \frac{2}{3}\sec\theta, \ \theta \in [0, \pi/2) \cup [\pi, 3\pi/2)$
$dx = \frac{2}{3}\sec\theta\tan\theta\, d\theta$
$\sqrt{\cdots} = \frac{2}{3}\tan\theta$

$= \int \frac{\cancel{9}}{\cancel{2\sec\theta}} \cdot \frac{2}{\cancel{3}}\tan\theta \cdot \frac{2}{\cancel{3}}\cancel{\sec\theta}\tan\theta\, d\theta$

$= 2\int \tan^2\theta\, d\theta = 2\int(\sec^2\theta - 1)d\theta$

$= 2(\tan\theta - \theta) + C$

$= 2\left(\frac{\sqrt{9x^2-4}}{2} - \sec^{-1}\left(\frac{3x}{2}\right)\right) + C$

$= \boxed{\sqrt{9x^2-4} - 2\sec^{-1}\left(\frac{3x}{2}\right) + C}$

$\sec\theta = \frac{3x}{2}$