SERIES - Extra Problems
2. Sections 11.2 – 11.4

This homework covers sections:

- 11.2 (Series; Test for Divergence; Geometric Series; Harmonic Series)
- 11.3 (Integral Test; p-series)
- 11.4 (Comparison Test, Limit Comparison Test).

I. Does the geometric series:

$$7 + 6 + \frac{36}{7} + \frac{216}{49} + \dots$$

converge or diverge? If it converges, what does it converge to?

II. For what values of x does the series:

$$\sum_{n=1}^{\infty} (-6)^n x^n$$

converge? For those values of x, what does the series converge to?

III. Suppose the sequence $\{s_n\}$ of partial sums of a series $\sum a_n$ is given by:

$$s_n = \frac{2n-1}{n+1}$$

.

a). Find the sequence a_n.

b). Find $\sum_{n=1}^{\infty} a_n$, if this converges.

IV. What are the conclusions of the Integral Test for the series:

$$\sum_{n=1}^{\infty} \frac{\tan(2n)}{6n^2+1}$$

V. Does the Integral Test apply to the series:

$$\sum_{n=1}^{\infty} \frac{1}{n \ln(n)}$$

? If so, what is the conclusion of this test?

VI. a). Does the Integral Test apply to the series:

$$\sum_{n=1}^{\infty} e^{-2n}$$

? If so, what is its conclusion?
b). What kind of series is the series above? Is there some other way to tell if it converges or not, besides the Integral Test? If it is convergent, can you tell what the series converges to?

VII. Give an example of a *divergent* series $\sum_{n=1}^{\infty} a_n$ with $\lim_{n\to\infty} a_n = 0$. Give an example of a *convergent* series $\sum_{n=1}^{\infty} a_n$ with $\lim_{n\to\infty} a_n = 0$. Explain what this illustrates about the Test for Divergence.

VIII. For each of the following series, determine whether they are convergent or divergent. Carefully explain your reasoning.

1).

$$\sum_{n=1}^{\infty} (\cos(25))^n$$

2).

$$\sum_{n=1}^{\infty} \frac{9+7^n}{6^n}$$

3).

$$\sum_{n=1}^{\infty} \arctan(10n)$$

4).

$$\sum_{n=1}^{\infty} \frac{2+\sin(n)}{9^n}$$

5).

$$\sum_{n=1}^{\infty} \frac{2n^2}{3n^6+2n+1}$$

6).

$$\sum_{n=1}^{\infty} \frac{2n^5}{3n^6+2n+1}$$

7).

$$\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{20}+\dots$$

8).

$$\sum_{n=1}^{\infty} \frac{1+5^n}{8^n}$$

9).

$$\sum_{n=1}^{\infty} \ln\left(\frac{n^2+6}{4n^2+3}\right)$$

10).

$$\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^8+8}$$

11).

$$\sum_{n=1}^{\infty} \frac{\arctan(200n)}{n^{1.001}}$$

12).

$$\sum_{n=1}^{\infty} \frac{6n^2(6n-1)!}{(6n+1)!}$$

13).

$$\sum_{n=1}^{\infty} \frac{n}{4n^3+2}$$

14).

$$\sum_{n=1}^{\infty} \frac{n^4}{5n^5-4}$$

15).

$$\sum_{n=1}^{\infty} \frac{n^4}{5n^5+4}$$

16).

$$\sum_{n=1}^{\infty} \frac{5^n}{(-3)^{n-1}}$$

17).

$$\sum_{n=1}^{\infty} \frac{7^{n+1}}{6^n-4}$$

18).

$$1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\frac{1}{125}+\dots$$

19).

$$\sum_{n=1}^{\infty} \frac{1}{3n+5}$$

20).

$$1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\dots$$

21).

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$$

22).

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\dots$$