SERIES - Extra Problems
4. Sections 11.8 – 11.10

1). Find the radius of convergence and the interval of convergence of the power series:

a).

$$\sum_{n=1}^{\infty} 5(-1)^n n x^n$$

b).

$$\sum_{n=1}^{\infty} \frac{x^n}{3n-1}$$

c).

$$\sum_{n=1}^{\infty} \frac{x^{n+2}}{3n!}$$

d).

$$\sum_{n=1}^{\infty} \frac{7^n \cdot x^n}{n^3}$$

e).

$$\sum_{n=0}^{\infty} \frac{(x-4)^n}{n^5+1}$$

f).

$$\sum_{n=1}^{\infty} n!(x+2)^n$$

g).

$$\sum_{n=1}^{\infty} \frac{n}{2^n}(x+2)^n$$

2). Suppose $\sum_{n=1}^{\infty} c_n 5^n$ converges. What can you conclude about:

a).

$$\sum_{n=1}^{\infty} c_n(-2)^n$$

b).

$$\sum_{n=1}^{\infty} c_n(-5)^n$$

3). Suppose $\sum_{n=1}^{\infty} c_n(-4)^n$ converges, and $\sum_{n=1}^{\infty} c_n 6^n$ diverges. What can you conclude about:

a).

$$\sum_{n=1}^{\infty} c_n$$

b).

$$\sum_{n=1}^{\infty} c_n 8^n$$

c).

$$\sum_{n=1}^{\infty} c_n(-2)^n$$

d).

$$\sum_{n=1}^{\infty} c_n 9^n$$

e).

$$\sum_{n=1}^{\infty} c_n(-6)^n$$

f).

$$\sum_{n=1}^{\infty} c_n 5^n$$

4). Use the geometric series to express the following functions as power series. For each one, determine the radius and interval of convergence:

a).

$$f(x) = \frac{1}{x+8}$$

b).

$$f(x) = \frac{6}{7-x}$$

c).

$$f(x) = \frac{x}{4+x^2}$$

d).

$$f(x) = \frac{x^2}{1-x^3}$$

5). Use the fact that:

$$\ln(3-x) = -\int \frac{1}{3-x}dx$$

to find a power series representation for the function:

$$f(x) = \ln(3-x)$$

Find the radius of convergence.

6). Evaluate the integral as a power series:

$$\int \frac{x}{1-x^{11}}dx$$

7). a). Starting with the geometric series, find the sum of the series:

$$\sum_{n=1}^{\infty} nx^{n-1};\ |x| < 1.$$

b). Use the result above to find:

$$\sum_{n=1}^{\infty} nx^{n};\ |x| < 1.$$

c). Use the result above to find:

$$\sum_{n=1}^{\infty} \frac{n}{7^n}$$

8). If $f^{(n)}(0) = (n+1)!$ for $n = 0, 1, 2, \ldots$ find the Maclaurin series of f and find its radius of convergence.

9). If $f^{(n)}(2) = \frac{(-1)^n n!}{8^n(n+3)}$ for $n = 0, 1, 2, \ldots$ find the Taylor series of f centered at 2 and find its radius of convergence.

10). Use one of the Taylor series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \text{ for all real } x.$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \text{ for all real } x.$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \text{ for all real } x.$$

to find the Maclaurin series for the functions:

a).

$$f(x) = \sin\left(\frac{\pi x^2}{5}\right)$$

b).

$$f(x) = e^{-5x}$$

c).

$$f(x) = 4e^x + e^{6x}$$

11). Evaluate:

$$\int \frac{e^x - 1}{2x} dx$$

as a power series.

12). Evaluate:

$$\int \frac{\cos(x) - 1}{x} dx$$

as a power series.