① $\int 2x\sqrt{x^2+1}\,dx$ Substitution: $u = x^2+1$, $du = 2x\,dx$

$= \int \sqrt{u}\,du = \frac{2}{3}u^{3/2}$

$= \frac{2}{3}(x^2+1)^{3/2} + C$

② $\int \frac{1}{(x+1)^2}dx = -\frac{1}{x+1} + C$ Useful: $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

③ $\int \sin(2x-4)dx = -\cos(2x-4)\cdot\frac{1}{2} + C$

④ $\int \frac{x+1}{(x^2+2x)^3}dx$ $u = x^2+2x$, $du = (2x+2)dx = 2(x+1)dx$

$\frac{x+1}{(x^2+2x)^3}dx \to \frac{1}{2}du$

$= \int \frac{1}{2}\frac{du}{u^3} = \frac{1}{2}\left(-\frac{1}{2}\right)u^{-2} + C = -\frac{1}{4}\frac{1}{u^2} + C = -\frac{1}{4(x^2+2x)^2} + C$

⑤ $\int \sqrt{4x-1}\,dx$ $u = 4x-1$, $du = 4dx$

$dx \to \frac{1}{4}du$

$= \frac{1}{4}\int \sqrt{u}\,du = \frac{1}{4}\frac{2}{3}u^{3/2} + C = \frac{1}{6}(4x-1)^{3/2} + C$

⑥ $\int x^2\sqrt{4x-1}\,dx$ $u = 4x-1 \to x = \frac{1}{4}(u+1)$, $du = 4dx$

$\int \left(\frac{1}{4}(u+1)\right)^2\cdot\sqrt{u}\cdot\frac{1}{4}du = \frac{1}{64}\int (u^2+2u+1)\sqrt{u}\,du$

$= \frac{1}{64}\int \left(u^{5/2} + 2u^{3/2} + u^{1/2}\right)du$

$= \frac{1}{64}\left(\frac{2}{7}u^{7/2} + 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right) + C$

$= \frac{1}{224}(4x-1)^{7/2} + \frac{1}{160}(4x-1)^{5/2} + \frac{1}{96}(4x-1)^{3/2} + C.$

⑦ $\int xe^{-x^2}dx$ $u = -x^2$, $du = -2x\,dx$

$x\,dx \to -\frac{1}{2}du$

$= \int -\frac{1}{2}du\,e^u = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C.$

⑧ $\int \frac{e^t}{e^{2t}+2e^t+1}dt$ $u = e^t$, $du = e^t dt$

$= \int \frac{1}{u^2+2u+1}du = \int \frac{1}{(u+1)^2}du = -\frac{1}{1+u} + C = -\frac{1}{1+e^t} + C$

Trigonometric Functions:

$(\cos x)' = -\sin x$

$(\sin x)' = \cos x$

$(\tan x)' = \sec^2 x = \frac{1}{\cos^2 x}$ $\qquad$ $(\sec x)' = \sec x \tan x$

$(\cot x)' = -\csc^2 x = -\frac{1}{\sin^2 x}$ $\qquad$ $(\csc x)' = -\csc x \cdot \cot x.$

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(9). $\int (\sec^2\theta) e^{\tan\theta} d\theta$ $\qquad$ $u = \tan\theta$, $du = \sec^2\theta \, d\theta$

$= \int e^u du = e^u + C$

$= e^{\tan\theta} + C$

(10). $\int \sin^2 x \cdot \cos x \, dx$ $\qquad$ $u = \sin x$, $du = \cos x \, dx$

$= \int u^2 du$

$= \frac{u^3}{3} + C = \frac{1}{3}\sin^3 x + C$

(11). $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$ $\qquad$ $u = \cos x$, $du = -\sin x \, dx$

$= -\int \frac{1}{u} du = -\ln|u| + C = -\ln|\cos x| + C.$

(12). $\int \cos x \cdot \cos(\sin x) dx$ $\qquad$ $u = \sin x$, $du = \cos x dx$

$= \int \cos u \, du = \sin u + C$

$= \sin(\sin x) + C$

(13). $\int \tan^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \int \left(\frac{1}{\cos^2 x} - 1\right) dx$

$= \int (\sec^2 x - 1) dx$

$= \tan x - x + C$

14. $\int \sin^2 x\,dx$

Double Angle Formulas:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$$

$$\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$$

Write

$$\sin^2 x = \frac{1}{2}(1-\cos(2x))$$

$$\Rightarrow \int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos(2x))\,dx$$

$$= \frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right) + C.$$

15. $\int \cos^2 x\,dx = \int \frac{1}{2}(1+\cos(2x))\,dx$

$$= \frac{1}{2}\left(x + \frac{1}{2}\sin(2x)\right) + C.$$

16. $\int \sin^2 x \cos^2 x\,dx = \int (\sin x \cos x)^2\,dx$

$$= \int \left(\frac{1}{2}\sin(2x)\right)^2 dx = \frac{1}{4}\int \sin^2(2x)\,dx$$

$$= \frac{1}{4}\int \frac{1}{2}(1-\cos(4x))\,dx$$

$$= \frac{1}{8}\left(x - \frac{1}{4}\sin(4x)\right) + C.$$

17. $\int \tan^2 x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$$= \int u^2\,du$$

$$= \frac{u^3}{3} + C = \frac{1}{3}\tan^3 x + C.$$