NAME:

MATH 133 - Michigan State University
November 14th, 2017.

Quiz 10

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you. GRADE YOUR OWN QUIZ!

(1pt.) **1.** Determine if the series below converges or diverges. Justify your reasoning.

$$\sum_{n=1}^{\infty} \frac{1}{n-100}.$$

$$\left.\begin{array}{l} \frac{1}{n-100} > \frac{1}{n} \text{ for all } n \geq 101 \\ \sum_{n=1}^{\infty} \frac{1}{n} = \infty \text{ (divergent, harmonic series)} \end{array}\right\} \Rightarrow \underline{\text{divergent}} \text{ by the Comparison Test}$$

(2pts.) **2.** Determine if the series below converges or diverges. Justify your reasoning.

$$\sum_{n=1}^{\infty} \sin^2\left(\frac{\pi}{n}\right).$$

$a_n = \sin^2\left(\frac{\pi}{n}\right)$

$b_n = \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin^2\left(\frac{\pi}{n}\right)}{\frac{1}{n^2}} = \lim_{n\to\infty} \left(\frac{\sin\left(\frac{\pi}{n}\right)}{\frac{\pi}{n}}\right)^2 \cdot \pi^2 = \pi^2 \text{ because } \lim_{x\to 0} \frac{\sin x}{x} = 1.$$

$\Rightarrow$ <u>Convergent</u> by the Limit Comp. Test., since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a <u>convergent</u> p-series ($p=2$).

(2pt.) **3.** Find the radius and interval of convergence for the series

$$\sum_{n=0}^{\infty} (nx)^n.$$

<u>Root Test</u>: $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} (n|x|) = \begin{cases} \infty & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$\Rightarrow$ The series converges only for $x=0$

$$\boxed{\begin{array}{l} R = 0 \\ I = \{0\} \end{array}}$$

(2pts.) **4.** Find the radius and interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}.$$

<u>Ratio Test</u>: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{2^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n x^n} \right| = \lim_{n\to\infty} \frac{2}{n+1} |x| = 0 < 1$ for all x

$\Rightarrow$ The series converges for all $x \in \mathbb{R}$

$$\boxed{\begin{array}{l} R = \infty \\ I = (-\infty, \infty) \end{array}}$$

(2pts)

5. Find the sum of the series:

$$\sum_{n=1}^{\infty} \frac{4 \cdot 3^{n-1}}{2^{3n+1}}. = \sum_{n=1}^{\infty} \frac{4}{3} \cdot \frac{3^n}{8^n \cdot 2} = \frac{2}{3} \sum_{n=1}^{\infty} \left(\frac{3}{8}\right)^n$$

$$= \frac{2}{3} \cdot \frac{3}{8} \sum_{n=0}^{\infty} \left(\frac{3}{8}\right)^n$$

$$= \frac{1}{4} \frac{1}{1-\frac{3}{8}} = \frac{1}{4} \cdot \frac{8}{5} = \boxed{\frac{2}{5}}$$

(1pt.)

6. Determine if the series below converges or diverges. Justify your reasoning.

$$\sum_{n=1}^{\infty} \frac{1}{3^n - 2^n}.$$

Compare to $\sum_{n=1}^{\infty} \frac{1}{3^n}$ (convergent - geometric series w/ $r = \frac{1}{3}$)

LCT: $a_n = \frac{1}{3^n - 2^n}$; $b_n = \frac{1}{3^n}$

$$\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \lim_{n\to\infty} \frac{3^n}{3^n - 2^n} = \lim_{n\to\infty} \frac{1}{1 - \underbrace{\left(\frac{2}{3}\right)^n}_{\xrightarrow{n\to\infty} 0}} = 1$$

=> Series converges by LCT.