

NAME:

Quiz 12

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

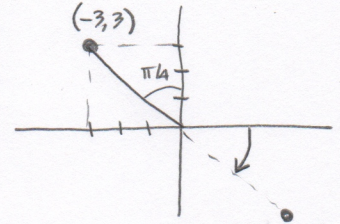
[2 pts]

1. Consider the Cartesian point $(x, y) = (-3, 3)$. a). Give a set of polar coordinates (r, θ) for this point, with $r > 0$.

$$r = \sqrt{x^2 + y^2} = \sqrt{2 \cdot 3^2} = 3\sqrt{2} \quad ; \quad \theta = \frac{\pi}{2} + \frac{\pi}{4}$$

(1 pts.)

$$\boxed{(3\sqrt{2}, \frac{3\pi}{4})}$$



b). Give a set of polar coordinates (r, θ) for this point, with $r < 0$.

(1 pts.)

$$\boxed{(-3\sqrt{2}, -\frac{\pi}{4})} \quad \text{or} \quad \boxed{(-3\sqrt{2}, \frac{7\pi}{4})}$$

[8 pts]

2. Consider the polar curves:

$$C_1 : r = 4 \sin \theta$$

$$C_2 : r = 2\sqrt{3}$$

a). Find the Cartesian equations of each curve.

$$C_1 : r^2 = 4r \sin \theta$$

$$x^2 + y^2 = 4y$$

$$x^2 + y^2 - 4y + 4 = 4$$

(2 pts.)

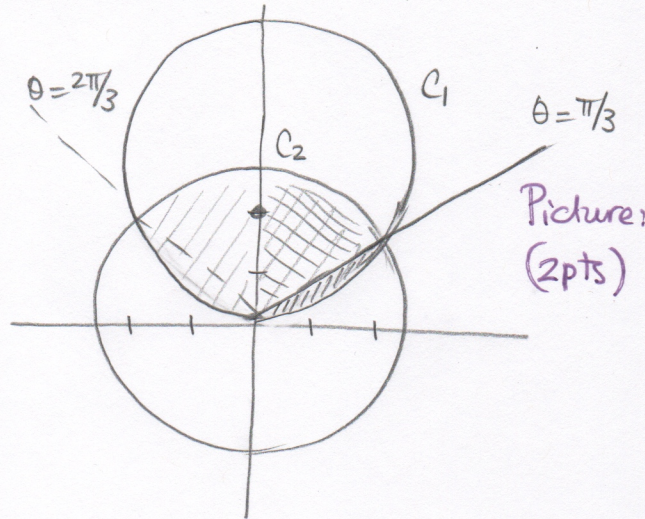
$$\boxed{x^2 + (y-2)^2 = 4}$$

radius = 2
center = (0, 2)

$$C_2 : r^2 = 12$$

$$\boxed{x^2 + y^2 = 12} \quad (1 \text{ pt.})$$

radius = $2\sqrt{3}$
center = (0, 0)



Picture:
(2 pts)

b). Sketch both of the curves in the Cartesian plane.

Intersection Angle:

$$4 \sin \theta = 2\sqrt{3}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

(1 pt.)

$$A = 2 \times \left[\int_0^{\pi/3} \frac{1}{2} (4 \sin \theta)^2 d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2} (2\sqrt{3})^2 d\theta \right]$$

(2 pts.)

c). Set up an integral to compute the area shared by the two curves (do not evaluate integral).