Math 1550 (30)

Final Exam Study Guide

The Final Exam (worth 20% of your grade) will be on Thursday, December 9th, from 5:30 - 7:30PM. These problems should be a guide to prepare for the test.

Sections you should focus on:

- **Chapter 2:** Sections 2.1 through 2.6
- **Chapter 3:** Sections 3.1 through 3.7, and 3.10
- **Chapter 4:** Sections 4.2, 4.3, 4.4, 4.7, 4.9
- **Chapter 5:** Sections 5.2, 5.3, 5.4, 5.6, 5.7
- **Chapter 6:** Sections 6.1, 6.2 (just average value), 6.3, 6.4
- **Chapter 8:** Section 8.1.

Chapter 2 Problems

1. For what value of c is the function below continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 4 & \text{, if } \quad x < 2 \\ cx^2 - 1 & \text{, if } \quad x \geq 2 \end{cases}$$

2. Find the limits:

(a) $\lim_{x \to 4} \dfrac{x^2 - 16}{x - 4}$

(b) $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{3(x - 4)}$

(c) $\lim_{x \to 0} \dfrac{\sin(8x)}{x}$

(d) $\lim_{x \to 0+} \dfrac{\sin(5x)}{|x|}$

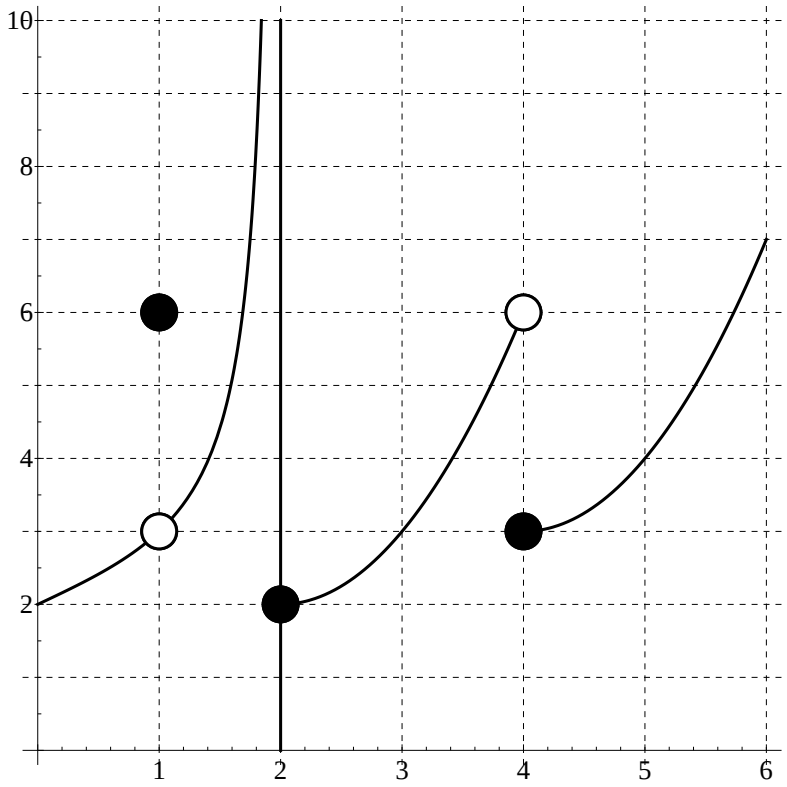
(e) $\lim_{x \to 0-} \dfrac{\sin(3x)}{|x|}$

(f) $\lim_{x \to 3} \dfrac{x - 3}{x^3 - 9x}$

(g) $\lim_{x \to 0} \dfrac{\sqrt{5 + x} - \sqrt{5 - x}}{x}$

(h) $\lim_{x \to 0} \dfrac{\tan(4x)}{\tan(8x)}$;

3. Below is the graph of a function f:



Find:

(a) $\lim_{x \to 1-} f(x)$

(b) $\lim_{x \to 1+} f(x)$

(c) $\lim_{x \to 2-} f(x)$

(d) $\lim_{x \to 2+} f(x)$

(e) $\lim_{x \to 4-} f(x)$

(f) $\lim_{x \to 4+} f(x)$

State the type of dicontinuity that f has at the points $x = 1$, $x = 2$, and $x = 4$.

Chapter 3 Problems

1. Find all the values of x where the tangent lines to $y = \frac{x^3}{3} + 3x$ and $y = 2x^2$ are parallel.

2. Determine the coefficients a and b such that $p(x) = x^2 + ax + b$ satisfies $p(1) = 4$ and $p'(0) = 1$.

3. Suppose that:

$$f(x+h) - f(x) = -5hx^2 + 6hx + 3h^2x - 7h^2 + 12h^3$$

   Find $f'(x)$.

4. Consider the functions f and g for which $f(0) = 3$, $g(0) = 1$, $f'(0) = -2$ and $g'(0) = 5$. Find $h'(0)$ for the function $h(x) = \frac{f(x)}{g(x)}$.

5. Suppose that:

$$f(x) = \frac{e^x}{x^2 + 1}$$

   Find $f'(1)$.

6. If $f(x) = \sqrt{x}$, find $f''(1)$.

7. Find $f'(x)$ for $f(x) = 5e^x \cos(x)$.

8. Find the equation for the line tangent to the graph of

$$f(x) = -4xe^x$$

   at the point $(a, f(a))$ for $a = 1$

9. Find the equation for the line tangent to the graph of

$$f(x) = \frac{5x}{x+4}$$

   at the point $(1, 1)$. Give the equation in the form $y = mx + b$.

10. Find the equation for the line tangent to the graph of

$$f(x) = \sqrt{x}$$

    at the point $(3, \sqrt{3})$.

11. Find $\frac{dy}{dx}$ for:

    (a) $y = x^{5x}$

    (b) $y = x^{3^x}$

12. Find the derivative $\frac{dy}{dx}$ for the following functions:

(a)
$$y = (x^2 - 1 + 2x^4)\cos(x)$$

(b)
$$y = 5e^x \sin(x)$$

(c)
$$y = x^5 \cos(x)$$

(d)
$$y = \frac{5x}{\sin(x) + \cos(x)}$$

(e)
$$y = \tan^{-1}(\cos(4x))$$

(f)
$$y = 2x \arcsin(x)$$

(g)
$$y = 6 \arcsin(x^2)$$

(h)
$$y = \sqrt{3x + \sqrt{5x}}$$

(i)
$$y = (x + \sin(x))^3$$

(j)
$$y = e^{8-x^2}$$

(k)
$$y = \cos(\sin(x))$$

(l)
$$y = 7e^{x \sin(x)}$$

(m)
$$y = -2\cos(\cos(x^3))$$

(n)
$$y = 3\ln(\sin(x) + \cos(x))$$

(o)
$$y = 5^{x^2 - x^4 + 3}$$

(p)
$$y = 4^{\sin(x^2)}$$

(q)
$$y = \frac{\ln(3x)}{\sin(x)}$$

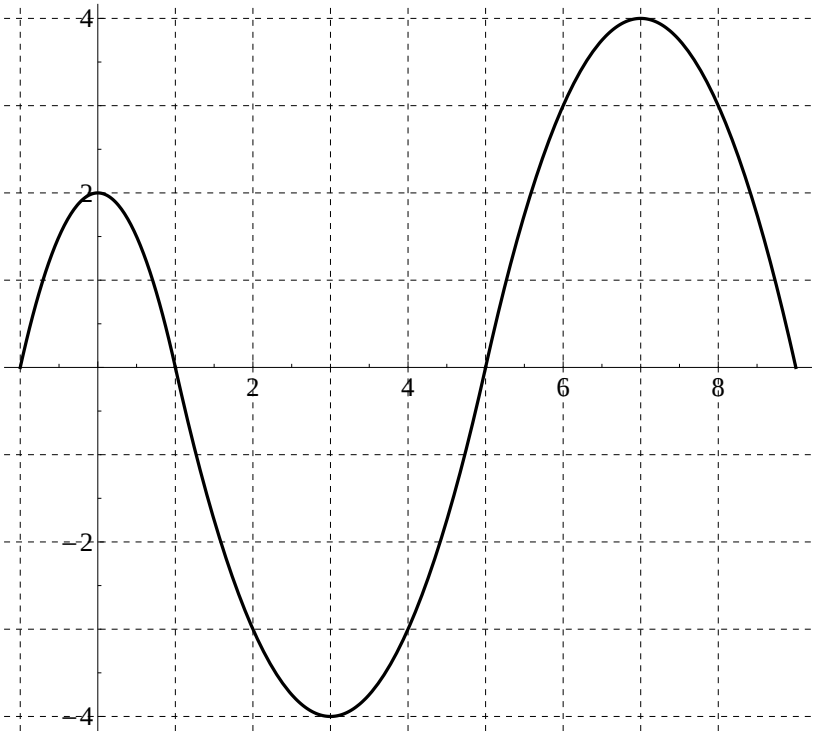
(r)
$$y = x\ln(x) - x$$

(s)
$$y = \ln(\cos(\sin(x)))$$

Chapter 4 Problems

1. Find the critical points of $f(x) = x\ln(4x)$.

2. Given below is the graph of the **derivative** f' of a function f.



   Use this graph to find:

   (a) The interval(s) where f is increasing:

   (b) The interval(s) where f is decreasing:

   (c) The interval(s) where f is concave up:

   (d) The interval(s) where f is concave down:

3. Find the minimum and the maximum of the function

$$f(x) = 2x^2 - 4x$$

   on the closed interval $[-1, 2]$.

4. Find the critical values of $f(x) = xe^{2x}$.

5. Find the positive critical point of:

$$f(x) = \frac{x}{x^{10} + 6}$$

6. Find the critical point of $f(x) = 3x - 15\ln(3x)$ (where $x > 0$), and determine if f has a local minimum or maximum at this point.

7. Find the inflection points of $f(x) = (x^2 + 3)e^x$.

8. Find the limits:

   (a)
   $$\lim_{x\to 0} \frac{11^x - 3^x}{x}$$

   (b)
   $$\lim_{x\to 0} \frac{x^2}{1 - \cos(x)}$$

   (c)
   $$\lim_{x\to 0} \frac{\sin(5x)}{\tan(3x)}$$

   (d)
   $$\lim_{x\to 0} \frac{e^x - 1}{\sin(4x)}$$

   (e)
   $$\lim_{x\to 1} \frac{e - e^x}{\ln(x)}$$

9. Find
   $$\lim_{x\to\infty} \left(\frac{14x}{14x + 6}\right)^{4x}$$

10. Given that $f''(x) = 4x + 3$, that $f'(0) = 1$, and $f(0) = 3$, find $f'(x)$ and $f(x)$.

11. Given that $f'(x) = 2e^x - 5$, and that $f(0) = 5$, find $f(x)$.

12. Find constants c_1 and c_2 such that
    $$F(x) = c_1 x\sin(x) + c_2\cos(x)$$
    is an antiderivative of
    $$f(x) = 2x\cos(x) + 4\sin(x)$$

Chapter 5 Problems

1. Use elementary geometry to find:

   (a)
   $$\int_0^6 \sqrt{36 - x^2}dx$$

   (b)
   $$\int_{-7}^7 \sqrt{49 - x^2}dx$$

   (c)
   $$\int_{-3}^3 \sqrt{9 - x^2}dx$$

2. Assuming that $\int_0^1 f(x)dx = 8$, $\int_0^2 f(x)dx = 10$, and $\int_1^4 f(x)dx = 10$, find:

   (a)
   $$\int_1^2 f(x)dx$$

   (b)
   $$\int_4^2 f(x)dx$$

3. Find
   $$\int_0^1 \left(\frac{d}{dt}\sqrt{3 + t^4}\right) dt$$

4. Find
   $$\int_0^1 \left(\frac{d}{dx} \ln\left(\frac{4x + 1}{2x + 4}\right)\right) dx$$

5. If
   $$f(x) = \int_x^{x^2} tdt$$
   find $f'(x)$.

6. If
   $$f(x) = \int_{\sqrt{x}}^x t^2 dt$$
   find $f'(x)$.

7. If
   $$f(x) = \int_4^{x^4} \sqrt{t^2 + 9}dt$$
   find $f'(x)$.

8. If
$$f(x) = \int_1^{\sqrt{x}} \ln(t^2)dt$$
find $f'(x)$.

9. If
$$f(x) = \begin{cases} x^2 & \text{if} \quad x < 1 \\ x & \text{if} \quad x \geq 1 \end{cases}$$
then find:
$$\int_0^3 f(x)dx$$

10. If
$$f(x) = \begin{cases} x & \text{if} \quad x < 2 \\ \frac{1}{x} & \text{if} \quad x \geq 2 \end{cases}$$
then find:
$$\int_0^5 f(x)dx$$

11. Find the integrals:

(a)
$$\int_5^4 e^x dx$$

(b)
$$\int_{-1}^1 (10x^9 + 5x^5)dx$$

(c)
$$\int_1^2 \frac{1}{5x}dx$$

(d)
$$\int \frac{9}{1+x^2}dx$$

(e)
$$\int 4\sqrt{x}dx$$

(f)
$$\int \left(\frac{4}{\sqrt[3]{x}} - 3\sqrt[3]{x^2}\right)dx$$

(g)
$$\int x(8 + 3x^6)dx$$

(h)
$$\int \left(x^3 + 8 + \frac{5}{x^2+1}\right)dx$$

(i)
$$\int \frac{3}{5\sqrt{x}}dx$$

(j)
$$\int \sec^2(4x)dx$$

(k)
$$\int_0^{0.3} \frac{1}{\sqrt{1-x^2}}dx$$

(l)
$$\int_1^2 \frac{5x^2+3}{x^2}dx$$

12. Use the Substitution Method to find:

(a)
$$\int (3x-2)^4 dx$$

(b)
$$\int \frac{1}{\sqrt{3x-4}} dx$$

(c)
$$\int x\sqrt{x^2-4} dx$$

(d)
$$\int \frac{10x^9 - 7x^6}{(x^{10}-x^7)^4} dx$$

(e)
$$\int x(x+1)^5 dx$$

(f)
$$\int \sin^5(x)\cos(x) dx$$

(g)
$$\int \frac{\ln^6(x)}{x} dx$$

(h)
$$\int \sec(x)\tan(x)(2+\sec(x))^3 dx$$

(i)
$$\int \frac{e^x}{2e^x - 5} dx$$

(j)
$$\int \frac{-\sin(x)}{1+\cos^2(x)} dx$$

(k)
$$\int \frac{3\arcsin(x)}{\sqrt{1-x^2}} dx$$

(l)
$$\int \frac{\arctan^3(x)}{1+x^2} dx$$

(m)
$$\int_1^{e^3} \frac{1}{x\sqrt{\ln(x)}} dx$$

Chapter 6 Problems

1. Find the area of the region between $y = e^x$ and $y = e^{5x}$ over $[0, 1]$.
2. Find the volume of a cylinder inclined at an anle of $60°$, of height $h = 12$, and whose base is a circle of radius 3.
3. Find the average value of $f(x) = 2xe^{x^2}$ on the interval $[0, 2]$.
4. Find the volume of the solid obtained by rotating the region under the graph of $f(x) = x^2 - 3x$ about the x-axis over $[0, 3]$.
5. Find the volume of the solid obtained by rotating the region under the graph of $f(x) = e^x$ about the x-axis over $[0, 3]$.
6. Find the volume of the solid obtained by rotating the region enclosed by the graphs $y = \sqrt{x}$ and $y = x^2$ about the x-axis.

Chapter 8 Problems

1. Calculate the arc length of $y = 3x + 1$ over $[0, 9]$.
2. Compute the surface area of revolution of $y = 4x + 3$ about the x-axis over the interval $[0, 1]$.