NAME: Solutions

Version 1

MATH 172 Texas A&M 10/03/2019

Exam 1

Show your work! You may not use calculators, notes or books.

Problem	Possible Score	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	10	

TOTAL:

(12) **1.** Find the integral:

$$\int \frac{3x}{x^2+1}\,dx.$$

Substitution: $u = x^2+1$ (3pts)

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$$\int \frac{3x}{x^2+1}\,dx = \int \underbrace{\frac{3}{u}}_{\text{2pts.}} \cdot \underbrace{\frac{1}{2}du}_{\text{2pts.}} = \frac{3}{2}\int \frac{1}{u}\,du = \overbrace{\frac{3}{2}\ln|u|}^{\text{3pts.}} + C$$

$$= \underbrace{\boxed{\frac{3}{2}\ln(x^2+1) + C}}_{\text{2pts.}}$$

(12) **2.** Given the function

$$y(x) = \int_{\sqrt{x}}^{\pi/4} \theta^2 \sec^2(\theta)\,d\theta,$$

find its derivative $y'(x)$.

Rewrite y as $y(x) = -\int_{\pi/4}^{\sqrt{x}} \theta^2 \sec^2(\theta)\,d\theta$ Changing order w/ minus sign: (4pts.)

By FTC I: (& Chain Rule)

$$\boxed{y'(x) = \underbrace{-(\sqrt{x})^2 \sec^2(\sqrt{x})}_{\text{4pts.}} \cdot \underbrace{\frac{1}{2\sqrt{x}}}_{\text{4pts.}}}$$

3. Find the integral:

$$\int \frac{x-4}{(x-2)(x-3)}\,dx.$$

Partial Fractions:

$$\frac{x-4}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$x-4 = A(x-3) + B(x-2)$$

$x=3$: $-1 = B$

$x=2$: $-2 = -A \Rightarrow A = 2$

8 pts. for p.f. decomp.

$$\Rightarrow \int \frac{x-4}{(x-2)(x-3)}\,dx = \int\left(\frac{2}{x-2} - \frac{1}{x-3}\right)dx = \boxed{\underbrace{2\ln|x-2|}_{2\text{pts.}} - \underbrace{\ln|x-3|}_{2\text{pts.}} + C}$$

4. Determine if the following integral converges to a finite number (if so, find it), diverges to $+\infty$ or $-\infty$ (if so, which one?), or diverges because it does not exist.

$$\int_0^1 \frac{e^{1/x}}{x^2}\,dx.$$

U-Sub: $u = \frac{1}{x}$; $du = \frac{-1}{x^2}$

$-du = \frac{1}{x^2}$

(a) $\int \frac{e^{1/x}}{x^2}dx = \int e^u \cdot (-du)$

$= -e^u + C$

$= \boxed{-e^{1/x} + C}$ 4 pts.

Limit write-up: 1 pt.

$t \to 0+$ 2 pts

Plug-in: 2 pts.

∞ limit: 3 pts.

(b) $\int_0^1 \frac{e^{1/x}}{x^2}dx = \lim_{t\to 0+} \int_t^1 \frac{e^{1/x}}{x^2}dx$

$= \lim_{t\to 0+} -e^{1/x}\Big|_t^1$

$= \lim_{t\to 0+} \left(-e^1 + \underbrace{e^{1/t}}_{}\right) = \boxed{\infty}$ Divergent to $+\infty$

$e^{1/0+} \to e^{\infty} \to \infty$

5. Find the integral:

$$\int x\sec^2(x)\,dx. = x\tan(x) - \int \tan(x)\,dx$$

$$= \boxed{x\tan(x) + \ln|\cos(x)| + C}$$

(2) (2)

By Parts:

$u = x \qquad du = dx$

$dv = \sec^2(x)\,dx \qquad v = \tan(x)$

(2) pts. each

6. Find the integral:

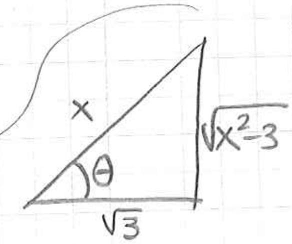
$$\int \frac{1}{(x^2-3)\sqrt{x^2-3}}\,dx. = \int \frac{1}{(\sqrt{3}\tan\theta)^2 \cdot \cancel{\sqrt{3}\tan\theta}}\,\sqrt{3}\sec\theta\,\cancel{\tan\theta}\,d\theta$$

(1) (1) (1)

Trig. Sub.: (2) $x = \sqrt{3}\sec\theta$, $\theta \in [0, \pi/2) \cup [\pi, 3\pi/2)$ (2)

$dx = \sqrt{3}\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-3} = \sqrt{3}\tan\theta$



$\frac{x}{\sqrt{3}} = \sec\theta$

$\frac{\sqrt{3}}{x} = \cos\theta$

$\sin\theta = \frac{\sqrt{x^2-3}}{x}$ (1)

$$= \int \frac{\sec\theta}{3\tan^2\theta}\,d\theta = \int \frac{\frac{1}{\cos\theta}}{3\,\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$= \frac{1}{3}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$\int \frac{1}{u^2}\,du = \frac{-1}{u} + C$

$$= \frac{-1}{3}\,\frac{1}{\sin\theta} + C \quad (2)$$

$$= \frac{-1}{3}\,\frac{1}{\frac{\sqrt{x^2-3}}{x}} + C = \boxed{\frac{-x}{3\sqrt{x^2-3}} + C} \quad (2)$$

7. Let R be the region in the xy-plane enclosed by the curves:

$$y = (x-2)^2; \quad y = x.$$

a). Find the points of intersection for these curves (fill in the boxes below):

(4)

The points of intersection are $(x, y) = \boxed{(1,1)}$ (2) and $(x, y) = \boxed{(4,4)}$ (2)

$(x-2)^2 = x$

$x^2 - 4x + 4 = x$

$x^2 - 5x + 4 = 0$

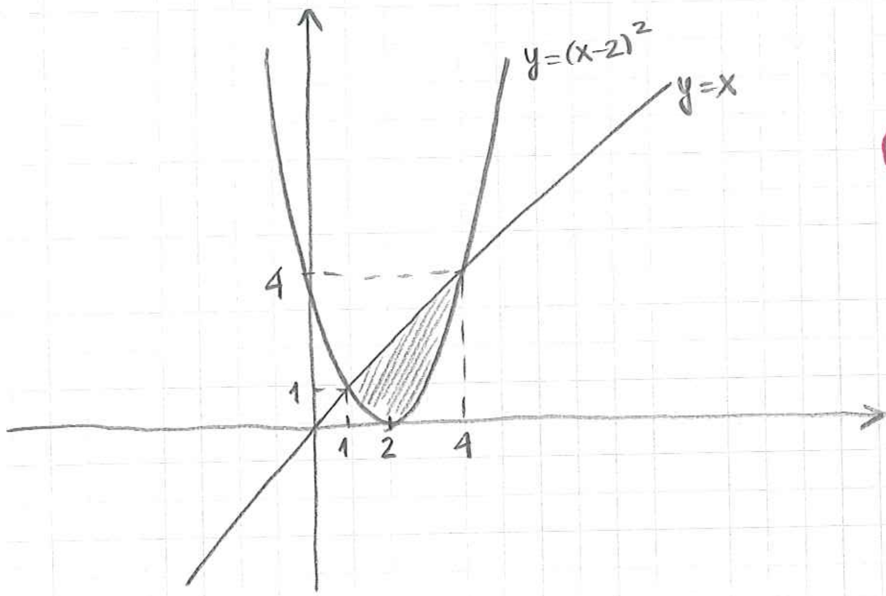
$(x-1)(x-4) = 0 \Rightarrow x = 1, 4$

$x = 1 \Rightarrow y = 1$

$x = 4 \Rightarrow y = 4$

b). Sketch the curves and the enclosed region R:

(4)



(4)

c). Fill in all the blanks below, to express the area of the region R as a definite integral (do not compute the numerical value; you do not have to simplify the expression of the function inside the integral):

(4)

(1) (2) (1)

$$\text{Area of } R = \int_{\boxed{1}}^{\boxed{4}} \boxed{x - (x-2)^2} \, d\boxed{x}$$

8. Let R be the region in the xy-plane enclosed by the curves:

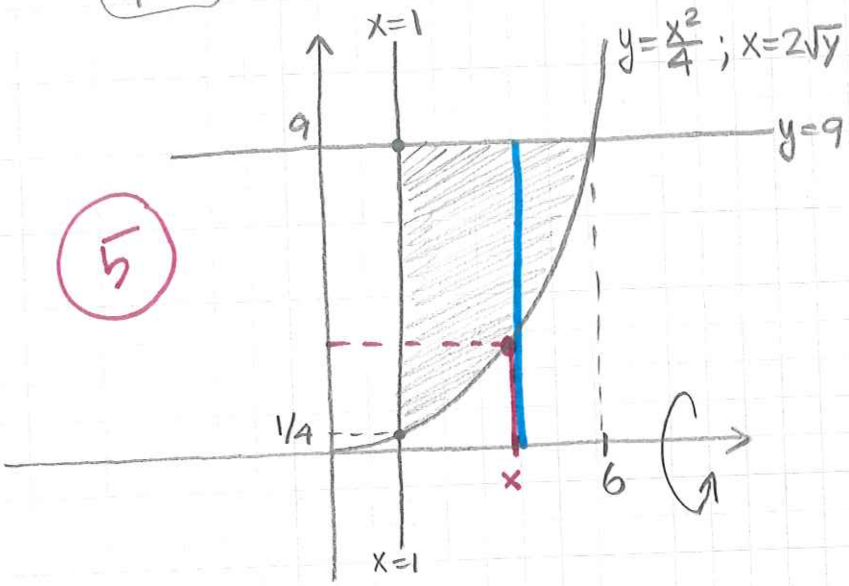
$$x = 2\sqrt{y};\quad y = 9;\quad x = 1.$$

a). Sketch the curves and the enclosed region R, and mark the intersection points on your graph:

$X = 2\sqrt{y} > 0$

$X^2 = 4Y$

$\boxed{\frac{X^2}{4} = Y}$ (but only half where $x>0$)!



$X = 2\sqrt{9} = 2\cdot 3 = 6$

5

Inner radius: $\frac{x^2}{4}$

Outer radius: 9

b). Fill in all the blanks below, to express the volume of the solid obtained by rotating the region R about the x-axis (do not compute the numerical value; you do not have to simplify the expression of the function inside the integral):

$$\text{Volume} = \int_{\boxed{1}}^{\boxed{6}} \boxed{\pi\left(9^2 - \left(\frac{x^2}{4}\right)^2\right)}\, d\,\boxed{x}$$

1 2 2 1 1

If it is helpful (it usually is), draw a 3D graph of the solid – however, this will not be graded.

5/

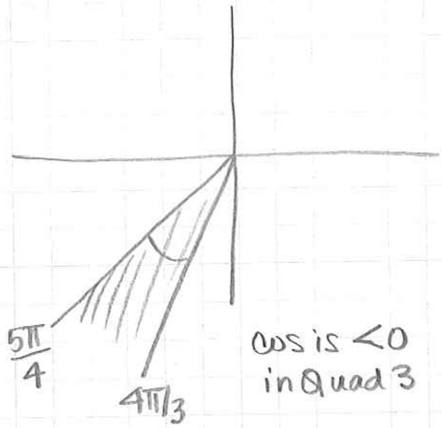
9. a). Find the integral (find the <u>numerical value</u>):

$$\int_{5\pi/4}^{4\pi/3} \sqrt{2+2\underbrace{\cos(2x)}_{\text{replace by } 2\cos^2(x)-1}}\,dx.$$

1

$$= \int_{5\pi/4}^{4\pi/3} \sqrt{2+2\left(2\cos^2(x)-1\right)}\,dx =$$

$$= \int_{5\pi/4}^{4\pi/3} \sqrt{4\cos^2(x)}\,dx = \int_{5\pi/4}^{4\pi/3} 2|\cos(x)| \quad 1$$



cos is <0 in Quad 3 1

$$= \int_{5\pi/4}^{4\pi/3} -2\cos(x)dx \;\; 1 \; = -2\sin(x)\Big|_{5\pi/4}^{4\pi/3} \quad 1$$

$$= -2\left(\sin\frac{4\pi}{3} - \sin\frac{5\pi}{4}\right) = -2\left(\frac{-\sqrt{3}}{2} - \frac{-\sqrt{2}}{2}\right) = 2\cdot\frac{\sqrt{3}-\sqrt{2}}{2} = \boxed{\sqrt{3}-\sqrt{2}}$$

5/ b). Find the integral:

$$\int (2x^2+1)e^{x^2}\,dx.$$

$$\int 2x^2e^{x^2}dx = \int x\cdot\left(2xe^{x^2}\right)dx \qquad \text{By Parts:}$$

$$= \int x\cdot\left(e^{x^2}\right)'dx$$

$$= xe^{x^2} - \int e^{x^2}dx. \qquad 5$$

$$\Rightarrow \int (2x^2+1)e^{x^2}dx = \left(xe^{x^2} - \cancel{\int e^{x^2}dx}\right) + \left(\cancel{\int e^{x^2}dx}\right)$$

$$= \boxed{xe^{x^2}+C}$$