NAME:

(*) Version 2

MATH 172 Texas A&M 10/03/2019

Exam 1

Show your work! You may not use calculators, notes or books.

Problem	Possible Score	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	10	

TOTAL:

1. Given the function

$$y(x) = \int_{\ln(x)}^{13} \sqrt[3]{1+t^2}\,dt,$$

find its derivative $y'(x)$.

Rewrite y as $y(x) = -\int_{13}^{\ln(x)} \sqrt[3]{1+t^2}\,dt \leftarrow$ 4pts

By FTC I + Chain Rule:

$$\boxed{y'(x) = \underbrace{-\sqrt[3]{1+\ln^2(x)}}_{\text{4pts.}} \cdot \underbrace{\frac{1}{x}}_{\text{4pts.}}}$$

2. Find the integral:

$$\int e^{2x} \cos\left(e^{2x}\right)\,dx.$$

Substitution: $u = e^{2x}$ 3pts.

$du = 2e^{2x}dx$

$\frac{1}{2}du = e^{2x}dx$

$$\int e^{2x}\cos(e^{2x})\,dx = \int \underbrace{\cos(u)}_{\text{2pts.}}\,\underbrace{\tfrac{1}{2}du}_{\text{2pts.}} = \overbrace{\tfrac{1}{2}\sin(u)+C}^{\text{3pts.}}$$

$$= \underbrace{\boxed{\tfrac{1}{2}\sin(e^{2x})+C}}_{\text{2pts.}}$$

3. Determine if the following integral converges to a finite number (if so, find it), diverges to $+\infty$ or $-\infty$ (if so, which one?), or diverges because it does not exist.

$$\int_0^9 \frac{e^{-1/x}}{x^2}\, dx.$$

(a) $\int \frac{e^{-1/x}}{x^2} dx \qquad u = -\frac{1}{x}\,;\; du = \frac{1}{x^2}dx$

$= \int e^u du = e^u + C$

$= \boxed{e^{-1/x} + C}$ (4pts.)

(b) $\int_0^9 \frac{e^{-1/x}}{x^2} dx = \lim_{t\to 0^+} \int_t^9 \frac{e^{-1/x}}{x^2} dx = \lim_{t\to 0^+} e^{-1/x}\Big|_t^9$

$= \lim_{t\to 0^+} \left(e^{-1/9} - \underbrace{e^{-1/t}}_{e^{-1/0+} \to e^{-\infty} \to 0}\right) = \boxed{e^{-1/9}}$

Limit write-up: (1pt.)
$t \to 0+$ (2pts.)
Plug-in: (2pts.)
Limit: (3pts.)

4. Find the integral:

$$\int \frac{x-5}{(x+1)(x-2)}\, dx.$$

$\frac{X-5}{(X+1)(X-2)} = \frac{A}{X+1} + \frac{B}{X-2}$

$X-5 = A(X-2) + B(X+1)$

$X=2: \quad -3 = 3B \quad B=-1$

$X=-1: \quad -6 = -3A \quad A=2$

(8pts.) for p.f. decomp.

$\Rightarrow \int \frac{X-5}{(X+1)(X-2)} dx = \int \left(\frac{2}{X+1} - \frac{1}{X-2}\right) dx = \boxed{\underbrace{2 \ln|X+1|}_{\text{2pts}} - \underbrace{\ln|X-2|}_{\text{2pts}} + C}$

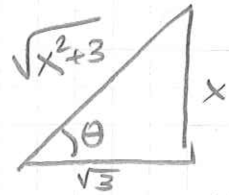
5. Find the integral:

$$\int \frac{1}{x^2\sqrt{x^2+3}}\,dx.$$

Trig. Sub.: $\overbrace{x=\sqrt{3}\tan\theta}^{2}$, $\underbrace{\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}_{2}$

$dx=\sqrt{3}\sec^2\theta\, d\theta$

$\sqrt{x^2+3}=\sqrt{3}\sec\theta$



$\tan\theta=\frac{x}{\sqrt{3}}$

$\Downarrow$

$\underbrace{\sin\theta=\frac{x}{\sqrt{x^2+3}}}_{1}$

$$\Rightarrow \int \frac{1}{x^2\sqrt{x^2+3}}\,dx = \int \frac{1}{\underbrace{3\tan^2\theta}_{1}\cdot\underbrace{\cancel{\sqrt{3}}\sec\theta}_{1}}\,\overbrace{\cancel{\sqrt{3}}\sec^2\theta\, d\theta}^{1}$$

$$= \int \frac{\sec\theta}{3\tan^2\theta}\,d\theta = \int \frac{\frac{1}{\cos\theta}}{\frac{3\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$= \int \frac{\cos\theta}{3\sin^2\theta}\,d\theta = \frac{-1}{3\sin\theta}+C \quad 2$$

$u=\sin\theta$

$du=\cos\theta\, d\theta$

$\int \frac{1}{3u^2}\,du=\frac{-1}{3u}+C$

$$= \frac{-1}{3\cdot\frac{x}{\sqrt{x^2+3}}}\,dx = \boxed{\frac{-\sqrt{x^2+3}}{3x}+C} \quad 2$$

6. Find the integral:

$$\int xe^{7x}\,dx.$$

By Parts: $u=x$ $\quad du=dx$

$dv=e^{7x}dx$ $\quad v=\frac{1}{7}e^{7x}$

} 2 pts. each

$$\Rightarrow \int xe^{7x}\,dx = \frac{1}{7}xe^{7x}-\int\frac{1}{7}e^{7x}\,dx$$

$$= \boxed{\underbrace{\frac{1}{7}xe^{7x}}_{\text{2pts.}}-\underbrace{\frac{1}{49}e^{7x}}_{\text{2pts.}}+C}$$

*

7. Let R be the region in the xy-plane enclosed by the curves:

$$x = 2\sqrt{y};\quad y = 9;\quad x = 1.$$

a). Sketch the curves and the enclosed region R, and mark the intersection points on your graph:

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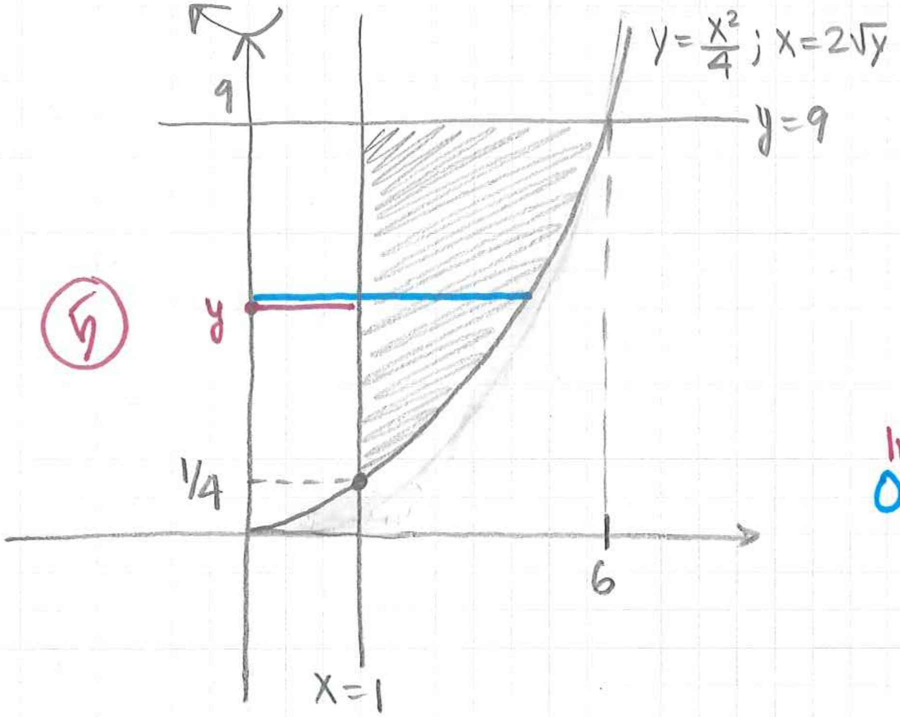


$x = 2\sqrt{y} > 0$

$\frac{x}{2} = \sqrt{y}$

$\frac{x^2}{4} = y$ (but only part where $x>0$)

$x = 2\sqrt{9} = 6$

Inner radius: 1

Outer radius: $x = 2\sqrt{y}$

b). Fill in all the blanks below, to express the volume of the solid obtained by rotating the region R about the **y-axis** (do not compute the numerical value; you do not have to simplify the expression of the function inside the integral):

1 2 2

$$\text{Volume} = \int_{\boxed{1/4}}^{\boxed{9}} \boxed{\pi\left((2\sqrt{y})^2 - 1^2\right)}\, d\boxed{y}$$

1 1

If it is helpful (it usually is), draw a 3D graph of the solid – however, this will not be graded.

8. Let R be the region in the xy-plane bounded by the curves:

$$y = 12 - x^2; \quad y = x.$$

a). Find the points of intersection for these curves (fill in the boxes below):

The points of intersection are $(x, y) = \boxed{(3,3)}$ and $(x, y) = \boxed{(-4,-4)}$

2 2

$12 - x^2 = x$

$x^2 + x - 12 = 0$

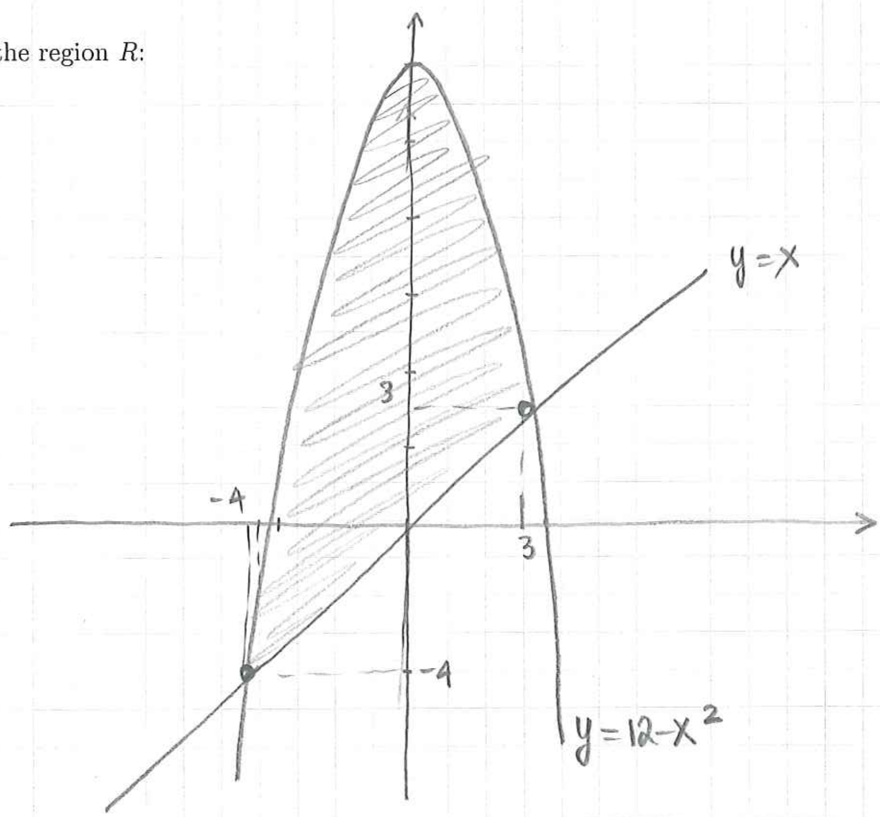
$(x+4)(x-3) = 0$

$x = -4, 3$

$x = -4 \Rightarrow y = -4$

$x = 3 \Rightarrow y = 3$

b). Sketch the region R:



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c). Fill in all the blanks below, to express the area of the region R as a definite integral (do not compute the numerical value; you do not have to simplify the expression of the function inside the integral):

$$\text{Area of } R = \int_{\boxed{-4}}^{\boxed{3}} \boxed{(12-x^2) - x} \, d\boxed{x}$$

1 1 2

9. a). Find the integral:

5/

$$\int \frac{x^{13}}{(1+x^7)^{1/3}}\, dx.$$

2

U-Sub: $u = 1 + x^7 \Rightarrow x^7 = u - 1$

$du = 7x^6 dx \Rightarrow x^6 dx = \frac{1}{7} du$

1

$$\Rightarrow \int \frac{x^{13}}{(1+x^7)^{1/3}} dx = \int \frac{x^7}{(1+x^7)^{1/3}} \cdot x^6 dx = \int \frac{u-1}{\sqrt[3]{u}} \cdot \frac{1}{7} du$$

1

$$= \frac{1}{7}\int \left(u^{2/3} - u^{-1/3}\right) du = \frac{1}{7}\left(\frac{3}{5}u^{5/3} - \frac{3}{2}u^{2/3}\right) + C$$

$$= \boxed{\frac{3}{35}(1+x^7)^{5/3} - \frac{3}{14}(1+x^7)^{2/3} + C}$$ 1

b). Find the integral:

5/

$$\int \frac{\ln(\tan(x))}{\sin(x)\cos(x)}\, dx.$$

$$\int \frac{\ln(\tan(x))}{\sin(x)\cos(x)} dx = \int \frac{\ln(\tan(x))}{\sin(x)} \cdot \frac{\cos(x)}{\cos^2(x)} dx$$

U-Sub

$u = \tan x$

$du = \sec^2 x\, dx$

$$= \int \ln(\tan(x)) \cdot \frac{1}{\tan(x)} \cdot \sec^2(x)\, dx$$

$$= \int \ln(u) \cdot \frac{1}{u} du$$

Another V-Sub:

$V = \ln(u)$

$dV = \frac{1}{u} du$

5

$$= \int v\, dv$$

$$= \frac{V^2}{2} + C$$

$$= \frac{\ln^2(u)}{2} + C$$

$$= \boxed{\frac{\ln^2(\tan(x))}{2} + C}$$