NAME:

Version 3

MATH 172 Texas A&M 10/03/2019

Exam 1

Show your work! You may not use calculators, notes or books.

Problem	Possible Score	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	10	

TOTAL:

1. Find the integral:

$$\int \frac{9x}{x^2+4}\, dx.$$

Substitution: $u = x^2+4$ (3pts.)

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$$\int \frac{9x}{x^2+4}\,dx = \int \underbrace{\frac{9}{u}}_{\text{(2pts.)}} \cdot \underbrace{\frac{1}{2}du}_{\text{(2pts.)}} = \frac{9}{2}\int \frac{1}{u}\,du = \overbrace{\frac{9}{2}\ln|u| + C}^{\text{(3pts.)}}$$

$$= \underbrace{\boxed{\frac{9}{2}\ln(x^2+4)+C}}_{\text{(2pts.)}}$$

2. Given the function

$$y(x) = \int_{\sqrt{x}}^{\pi} \theta^2 \sin^4(\theta)\, d\theta,$$

find its derivative $y'(x)$.

Rewrite: $y(x) = -\int_{\pi}^{\sqrt{x}} \theta^2 \sin^4(\theta)\,d\theta$ (4pts.)

FTC I + Chain Rule: $\boxed{y'(x) = \underbrace{-(\sqrt{x})^2 \sin^4(\sqrt{x})}_{\text{(4pts.)}} \cdot \underbrace{\frac{1}{2\sqrt{x}}}_{\text{(4 pts.)}}}$

3. Find the integral:

$$\int \frac{2x-5}{(x-1)(x-2)}\, dx.$$

$$\frac{2x-5}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$2x-5 = A(x-2)+B(x-1)$$

$$x=2: \ -1 = B$$

$$x=1: \ -3 = -A \Rightarrow A=3$$

8 pts. for p.f. decomp.

$$\Rightarrow \int \frac{2x-5}{(x-1)(x-2)}dx = \int\left(\frac{3}{x-1} - \frac{1}{x-2}\right)dx = \boxed{\underbrace{3\ln|x-1|}_{2\text{ pts.}} - \underbrace{\ln|x-2|}_{2\text{ pts.}} + C}$$

4. Determine if the following integral converges to a finite number (if so, find it), diverges to $+\infty$ or $-\infty$ (if so, which one?), or diverges because it does not exist.

$$\int_0^2 \frac{e^{1/x}}{x^2}\, dx.$$

(a) $\int \frac{e^{1/x}}{x^2}dx = \int -e^u du$ $\qquad u = \frac{1}{x}$; $du = \frac{-1}{x^2}dx$

$$= -e^u + C$$

$$= \boxed{-e^{1/x} + C} \quad 4\text{ pts.}$$

(b) $\int_0^2 \frac{e^{1/x}}{x^2}dx = \lim_{t\to 0+} \int_t^2 \frac{e^{1/x}}{x^2}dx = \lim_{t\to 0+} \left(-e^{1/x}\right)\Big|_t^2$

$$= \lim_{t\to 0+}\left(-e^{1/2} + \underbrace{e^{1/t}}_{e^{1/0+} \to e^{\infty} \to \infty}\right) = \boxed{\infty}$$ Divergent to $+\infty$

Limit write-up: 1 pt.
$t \to 0+$: 2 pts.
Plug-in: 2 pts.
do limit: 3 pts.

5. Find the integral:

$$\int x \sec^2(x)\, dx.$$

Same as #5 in <u>Version 1</u>

6. Find the integral:

$$\int \frac{1}{(x^2-8)\sqrt{x^2-8}}\, dx. = \int \frac{1}{(\sqrt{8}\tan\theta)^2 \cdot \cancel{\sqrt{8}\tan\theta}} \, \cancel{\sqrt{8}} \sec\theta \, \cancel{\tan\theta}\, d\theta$$

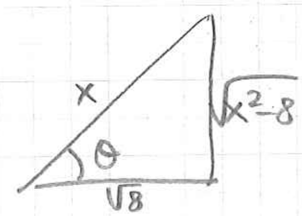
1 1 1

2 2

<u>Trig. Sub.</u>: $x = \sqrt{8}\sec\theta, \quad \theta \in [0, \pi/2) \cup [\pi, 3\pi/2)$

$dx = \sqrt{8}\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-8} = \sqrt{8}\tan\theta$



$\frac{x}{\sqrt{8}} = \sec\theta$

$\frac{\sqrt{8}}{x} = \cos\theta$

$$\sin\theta = \frac{\sqrt{x^2-8}}{x}$$ 1

$$= \int \frac{\sec\theta}{8\tan^2\theta}\, d\theta$$

$$= \frac{1}{8}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$\int \frac{1}{u^2}\, du = \frac{-1}{u} + C$

$$= \frac{-1}{8}\, \frac{1}{\sin\theta} + C$$ 2

$$= \frac{-1}{8}\, \frac{1}{\frac{\sqrt{x^2-8}}{x}} + C = \boxed{\frac{-x}{8\sqrt{x^2-8}} + C}$$ 2

7. Let R be the region in the xy-plane bounded by the curves:

$$y = (x-2)^2; \;\; y = x.$$

a). Find the points of intersection for these curves (fill in the boxes below):

The points of intersection are $(x, y) = \square$ and $(x, y) = \square$

b). Sketch the region R:

Same as #7 in Version 1

8. → Same as #8 in Version 1

c). Fill in all the blanks below, to express the area of the region R as a definite integral (do not compute the numerical value; you do not have to simplify the expression of the function inside the integral):

$$\text{Area of } R = \int_{\square}^{\square} \boxed{} \, d\square$$

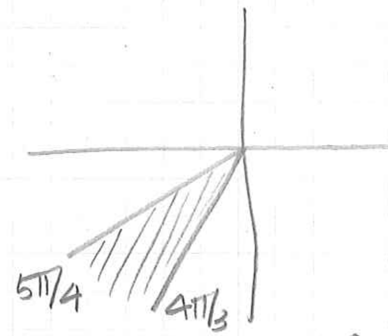
9. a). Find the integral (find the <u>numerical value</u>):

$$\int_{5\pi/4}^{4\pi/3} \sqrt{2 - 2\underbrace{\cos(2x)}_{}}\, dx. \qquad 1$$

Replace with $1-2\sin^2(x)$

$$= \int_{5\pi/4}^{4\pi/3} \sqrt{2-2\left(1-2\sin^2 x\right)}\,dx$$

$$= \int_{5\pi/4}^{4\pi/3} \sqrt{4\sin^2 x}\,dx = \int_{5\pi/4}^{4\pi/3} 2\,|\sin(x)|\,dx \qquad 1$$



sin is < 0 in Quad 3

$$= \int_{5\pi/4}^{4\pi/3} -2\sin(x)\,dx \qquad 1$$

$$= 2\cos(x)\Big|_{5\pi/4}^{4\pi/3} = 2\left(\cos\frac{4\pi}{3} - \cos\frac{5\pi}{4}\right) = 2\left(\frac{-1}{2} - \frac{-\sqrt{2}}{2}\right) = 2\cdot\frac{\sqrt{2}-1}{2} = \boxed{\sqrt{2}-1} \quad 1$$

1

b). Find the integral:

$$\int (2x^2+1)e^{x^2}\,dx.$$

Same as 9b in <u>Version 1.</u>