NAME:

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Version 4.

MATH 172 Texas A&M 10/03/2019

Exam 1

Show your work! You may not use calculators, notes or books.

Problem	Possible Score	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	10	

TOTAL:

1. Given the function

$$y(x) = \int_{\ln(x)}^{9} \sqrt[4]{1+t^3}\, dt,$$

find its derivative $y'(x)$.

Rewrite y as: $y(x) = -\int_{9}^{\ln(x)} \sqrt[4]{1+t^3}\,dt$ 4pts.

By FTCI + Chain Rule:

$$\boxed{y'(x) = -\underbrace{\sqrt[4]{1+\ln^3(x)}}_{4\text{pts.}} \cdot \underbrace{\frac{1}{x}}_{4\text{pts.}}}$$

2. Find the integral:

$$\int e^{3x} \sin\left(e^{3x}\right)\, dx.$$

Substitution: $u = e^{3x}$ 3 pts.

$du = 3e^{3x}dx$

$\frac{1}{3}du = e^{3x}dx$

$$\int e^{3x}\sin(e^{3x})\,dx = \int \underbrace{\sin(u)}_{2\text{pts.}} \cdot \underbrace{\tfrac{1}{3}du}_{2\text{pts.}} = \overbrace{-\tfrac{1}{3}\cos(u)+C}^{3\text{pts.}}$$

$$= \underbrace{\boxed{-\tfrac{1}{3}\cos(e^{3x})+C}}_{2\text{pts.}}$$

3. Determine if the following integral converges to a finite number (if so, find it), diverges to $+\infty$ or $-\infty$ (if so, which one?), or diverges because it does not exist.

$$\int_0^{10} \frac{e^{-1/x}}{x^2}\, dx.$$

(a) $\displaystyle\int \frac{e^{-1/x}}{x^2}\,dx = \int e^u\,du \qquad u = \frac{-1}{x};\ du = \frac{+1}{x^2}\,dx$

$= e^u + C = \boxed{e^{-1/x} + C}$ 4pts.

(b) $\displaystyle\int_0^{10} \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to 0+} \int_t^{10} \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to 0+} e^{-1/x}\Big|_t^{10}$

$\displaystyle = \lim_{t\to 0+} \left(e^{-1/10} - \underbrace{e^{-1/t}}_{e^{-1/0+} \to e^{-\infty} \to 0}\right) = \boxed{e^{-1/10}}$

Limit write-up: 1pt.
$t \to 0+$: 2pts.
Plug-in: 2 pts.
Limit: 3pts.

4. Find the integral:

$$\int \frac{x+6}{(x+1)(x-4)}\, dx.$$

$\dfrac{X+6}{(X+1)(X-4)} = \dfrac{A}{X+1} + \dfrac{B}{X-4}$

$X+6 = A(X-4) + B(X+1)$

$X=4:\ 10 = 5B \quad B=2$

$X=-1:\ 5 = -5A \quad A=-1$

8 pts. p.f. decomp.

$\displaystyle \Rightarrow \int \frac{X+6}{(X+1)(X-4)}\,dx = \int\left(\frac{-1}{X+1} + \frac{2}{X-4}\right)dx = \boxed{\underbrace{-\ln|X+1|}_{2\text{pts.}} + \underbrace{2\ln|X-4|}_{2\text{pts.}} + C}$

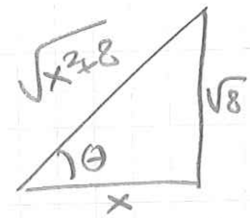
5. Find the integral:

$$\int \frac{1}{x^2\sqrt{x^2+8}}\,dx.$$

<u>Trig Sub</u>: $\underbrace{X = \sqrt{8}\tan\theta}^{2}$, $\underbrace{\theta \in (-\pi/2, \pi/2)}_{2}$

$dX = \sqrt{8}\sec^2\theta\,d\theta$

$\sqrt{X^2+8} = \sqrt{8}\sec\theta$



$\tan\theta = \frac{X}{\sqrt{8}}$

$\Downarrow$

$\underbrace{\sin\theta = \frac{X}{\sqrt{X^2+8}}}_{1}$

$$\Rightarrow \int \frac{1}{X^2\sqrt{X^2+8}}\,dx = \int \frac{1}{\underbrace{(\sqrt{8})^2\tan^2\theta}_{1} \cdot \underbrace{\sqrt{8}\sec\theta}_{1}} \overbrace{\sqrt{8}\sec^2\theta\,d\theta}^{1}$$

$$= \int \frac{\sec\theta}{8\tan^2\theta}\,d\theta = \int \frac{\frac{1}{\cos\theta}}{8\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$= \frac{1}{8}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta = \frac{-1}{8\sin\theta} + C \quad 2$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$\int \frac{1}{u^2}du = \frac{-1}{u} + C$

$$= \frac{-1}{8 \cdot \frac{X}{\sqrt{X^2+8}}}\,dx = \boxed{\frac{-\sqrt{X^2+8}}{8X} + C} \quad 2$$

6. Find the integral:

$$\int xe^{4x}\,dx.$$

By Parts: $U = X$ $\quad du = dX$

$dv = e^{4X}dX$ $\quad V = \frac{1}{4}e^{4X}$ $\Big\}$ 2 pts. each

$$\Rightarrow \int Xe^{4X}\,dx = \frac{X}{4}e^{4X} - \int \frac{1}{4}e^{4X}dX$$

$$= \boxed{\frac{X}{4}e^{4X} - \frac{1}{16}e^{4X} + C}$$

2pts. $\quad$ 2pts.

8. Let R be the region in the xy-plane bounded by the curves:

$$y = 12 - x^2; \ \ y = x.$$

a). Find the points of intersection for these curves (fill in the boxes below):

The points of intersection are $(x, y) = \boxed{}$ and $(x, y) = \boxed{}$

Same as #8 in version 2

b). Sketch the region R:

Same as #7. in Version 2

7. → Same as #7. in Version 2

c). Fill in all the blanks below, to express the area of the region R as a definite integral (do not compute the numerical value; you do not have to simplify the expression of the function inside the integral):

$$\text{Area of } R = \int_{\boxed{}}^{\boxed{}} \boxed{} \, d\boxed{}$$

9. a). Find the integral:

$$\int \frac{x^9}{(1+x^5)^{1/3}}\,dx.$$

U-Sub: 2 $u = 1+x^5 \Rightarrow u-1 = x^5$

$du = 5x^4\,dx \Rightarrow \frac{1}{5}du = x^4\,dx$

1

$$\int \frac{x^9}{(1+x^5)^{1/3}}\,dx = \int \frac{x^5}{(1+x^5)^{1/3}}\cdot x^4\,dx = \int \frac{u-1}{\sqrt[3]{u}}\cdot\frac{1}{5}du$$

1

$$= \frac{1}{5}\int \left(u^{2/3} - u^{-1/3}\right)du = \frac{1}{5}\left(\frac{3}{5}u^{5/3} - \frac{3}{2}u^{2/3}\right) + C$$

$$= \boxed{\frac{3}{25}(1+x^5)^{5/3} - \frac{3}{10}(1+x^5)^{2/3} + C}$$ 1

b). Find the integral:

$$\int \frac{\ln(\tan(x))}{\sin(x)\cos(x)}\,dx.$$

Same as 9b in Version 2