NAME: Solutions

MATH 172, Section 501; 09/18/2019

Quiz 3

Show your work! You may not use calculators, notes or books.

1. Find the integral

$$\int \frac{1}{x^2\sqrt{x^2+9}}\,dx.$$

[TURN PAGE for trig formulas!]

(1 pt.) $\rightarrow x = 3\tan\theta$, $\theta \in (-\pi/2, \pi/2)$ (1 pt.)

$dx = 3\sec^2\theta\, d\theta$ (1 pt.)

$\sqrt{x^2+9} = 3\sec\theta$ (1 pt.)

$$\int \frac{1}{x^2\sqrt{x^2+9}}\,dx = \int \frac{1}{(3\tan\theta)^2 \cdot 3\sec\theta} 3\sec^2\theta\, d\theta$$

reaching correct integral in θ: (1 pt.)

$$= \frac{1}{9}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{9}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

solving the integral in θ (i.e. reaching) (3 pts.)

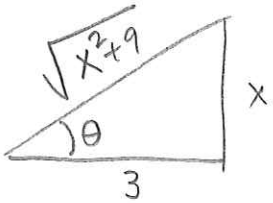
$$= \frac{1}{9}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta \qquad u = \sin\theta,\ du = \cos\theta\, d\theta$$

$$= \frac{1}{9}\int \frac{1}{u^2}\,du = \frac{-1}{9u} + C$$

$$= \frac{-1}{9\sin\theta} + C = \boxed{\frac{-\sqrt{x^2+9}}{9x} + C}$$

final answer in x: (2 pts.) (includes finding $\sin\theta$)

$\frac{x}{3} = \tan\theta$



$\sin\theta = \frac{x}{\sqrt{x^2+9}}$

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Show your work! You may not use calculators, notes or books.

1. Find the integral

$$\int \frac{\sqrt{x^2-16}}{x^3}\,dx.$$

[TURN PAGE for trig formulas!]

(1pt.) → $x = 4\sec\theta$, $\theta \in [0, \pi/2) \cup [\pi, 3\pi/2)$ ← (1pt.) [also OK if they write Quad I & III]

$dx = 4\sec\theta\tan\theta$ (1pt.)

$\sqrt{x^2-16} = 4\tan\theta$ (1pt.)

$$\int \frac{\sqrt{x^2-16}}{x^3}\,dx = \int \frac{4\tan\theta}{4^3\sec^3\theta}\,4\sec\theta\tan\theta\,d\theta$$

$$= \frac{1}{4}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta$$ ← reaching correct integral in θ: (1pt.)

$$= \frac{1}{4}\int \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}\,d\theta$$

solving integral in θ: (2pts.)

$$= \frac{1}{4}\int \sin^2\theta\,d\theta$$

$$= \frac{1}{4}\left(\frac{\theta}{2} - \frac{1}{4}\sin(2\theta)\right) + C$$

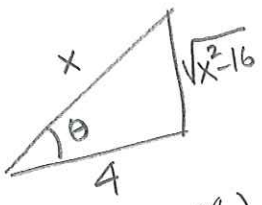
$$= \frac{\theta}{8} - \frac{1}{8}\sin\theta\cos\theta + C$$

$$= \frac{1}{8}\arccos\left(\frac{4}{x}\right) - \frac{1}{8}\cdot\frac{\sqrt{x^2-16}}{x}\cdot\frac{4}{x} + C$$

$$= \boxed{\frac{1}{8}\arccos\left(\frac{4}{x}\right) - \frac{\sqrt{x^2-16}}{2x^2} + C}$$ ← final answer in x: (3pts.) (includes finding $\sin\theta$, $\cos\theta$)

$\frac{x}{4} = \sec\theta = \frac{1}{\cos\theta}$

$\frac{4}{x} = \cos\theta$



$\theta = \arccos\left(\frac{4}{x}\right)$ OR $\operatorname{arcsec}\left(\frac{x}{4}\right)$ ← OK also

$\sin\theta = \frac{\sqrt{x^2-16}}{x}$

$\cos\theta = \frac{4}{x}$