NAME: Solutions

5pts. each

MATH 172, Section 501; 09/25/2019

Quiz 4

Show your work! You may not use calculators, notes or books.

1. Find the integral

$$\int \frac{x}{x^2+1}\,dx.$$

$$= \int \frac{1}{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln|x^2+1| + C}$$

Substitution: $u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

2. Find the integral

$$\int_1^\infty \frac{1}{(x+1)^3}\,dx$$

Improper

$$= \lim_{t\to\infty} \int_1^t \frac{1}{(x+1)^3}\,dx$$

$$= \lim_{t\to\infty} \left.\frac{-1}{2}(x+1)^{-2}\right|_1^t = \lim_{t\to\infty} \left.\frac{-1}{2(x+1)^2}\right|_1^t$$

$$= \lim_{t\to\infty} \left(\underbrace{\frac{-1}{2(t+1)^2}}_{\frac{-1}{\infty}\to 0} + \frac{1}{2\cdot 2^2}\right) = \frac{1}{8}$$

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3. Find the integral

$$\int \ln(\sqrt{x})\,dx.$$

By Parts: $u = \ln(\sqrt{x})$ $\quad du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2x}\,dx$

$dv = dx$ $\quad v = x$

$$\int \ln(\sqrt{x})\,dx = x\ln(\sqrt{x}) - \int x \cdot \frac{1}{2x}\,dx$$

$$= x\ln(\sqrt{x}) - \int \frac{1}{2}\,dx$$

$$= \boxed{x\ln(\sqrt{x}) - \frac{1}{2}x + C}$$

4. Find the integral

$$\int \frac{1}{x^3\sqrt{x^2-4}}\,dx.$$

Trig Sub: $x = 2\sec\theta$, $\theta \in [0, \pi/2) \cup [\pi, 3\pi/2)$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-4} = 2\tan\theta$

$$\int \frac{1}{x^3\sqrt{x^2-4}}\,dx = \int \frac{1}{2^3\sec^3\theta \cdot 2\tan\theta}\, 2\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{8\sec^2\theta}\,d\theta = \frac{1}{8}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{8}\left(\frac{\theta}{2} + \frac{1}{4}\underbrace{\sin(2\theta)}_{2\sin\theta\cos\theta}\right) + C$$

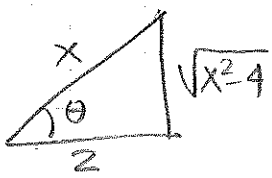
$$= \frac{1}{8}\left(\frac{\theta}{2} + \frac{1}{2}\sin\theta\cos\theta\right) + C$$

$$= \boxed{\frac{1}{16}\left(\sec^{-1}\left(\frac{x}{2}\right) + \frac{\sqrt{x^2-4}}{x}\cdot\frac{2}{x}\right) + C}$$

or $\cos^{-1}\left(\frac{2}{x}\right)$

$\sec\theta = \frac{x}{2}$

$\cos\theta = \frac{2}{x}$



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Show your work! You may not use calculators, notes or books.

1. Find the integral

$$\int \frac{\ln(x)}{x^2}\,dx.$$

By Parts: $u = \ln x$ $\quad du = \frac{1}{x}dx$

$dv = \frac{1}{x^2}dx$ $\quad v = -\frac{1}{x}$

$$\int \frac{\ln(x)}{x^2}dx = \frac{-1}{x}\ln x - \int \frac{-1}{x}\cdot\frac{1}{x}dx$$

$$= -\frac{1}{x}\ln x - \int \frac{-1}{x^2}dx$$

$$= \boxed{\frac{-1}{x}\ln x - \frac{1}{x} + C}$$

2. Find the integral

$$\int_1^\infty e^{-3x}\,dx$$ Improper

$$= \lim_{t\to\infty}\int_1^t e^{-3x}dx$$

$$= \lim_{t\to\infty} \frac{-1}{3}e^{-3x}\Big|_1^t$$

$$= \lim_{t\to\infty}\left(\underbrace{\frac{-1}{3}e^{-3t}}_{e^{-\infty}\to 0} + \frac{1}{3}e^{-3}\right) = \boxed{\frac{1}{3e^3}}$$

(#3) $\int e^x \sin(e^x)\,dx$ Substitution: $u = e^x$, $du = e^x dx$

$$= \int \sin(u)\,du$$

$$= -\cos(u) + C = \boxed{-\cos(e^x) + C}$$

(#4) Same as #4 in Q4-S501.

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