

NAME: Solutions

MATH 172 10/09/2019

Quiz 5

Show your work! You may not use calculators, notes or books.

1. Given each sequence $\{a_n\}_{n \in \mathbb{N}}$ below, find $\lim_{n \rightarrow \infty} a_n$ (You can just write down the answer in the box, no need to justify):

a). $a_n = \frac{3 + 5n^2}{n + n^2}$; $\lim_{n \rightarrow \infty} a_n = \boxed{5}$ 3 pts.

b). $a_n = \frac{n^2}{\sqrt{n^3 + 2n + 1}}$; $\lim_{n \rightarrow \infty} a_n = \boxed{\infty}$ 3 pts.

2. Find the limit below:

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(1 + \frac{4}{n^2}\right)^{n^2} &= \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2/4}\right)^{n^2} && 1 \text{ pt.} \\ &= \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n^2/4}\right)^{n^2/4} \right]^4 \cdot n^2 \xrightarrow{n \rightarrow \infty} 4 && 1 \text{ pt.} \\ &\quad \downarrow \text{ } n \rightarrow \infty \\ &\quad e && \\ &= \boxed{e^4} && 2 \text{ pts.} \end{aligned}$$