

NAME: *Solutions*

MATH 172 10/16/2019

Quiz 6

Show your work!

Explain where is the error in the following reasoning:

$$\begin{aligned} 0 &= 0 + 0 + 0 + \dots \\ &= (1 - 1) + (1 - 1) + (1 - 1) + \dots \\ \textcircled{=} & 1 - 1 + 1 - 1 + 1 - 1 + \dots \\ &= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \dots \\ &= 1 + 0 + 0 + 0 + \dots \\ &= 1. \end{aligned}$$

Background: The mathematician Guido Grandi (1671 - 1742) believed that this computation proved the existence of God, because "something has been created out of nothing."

An important lesson to learn here for series in general is this: it is true that for FINITE sums, the order in which you add numbers does not matter:

$$1 + 2 - 4 + 3 = (1 + 2) + (-4 + 3) = 1 + (2 - 4) + 3 = (1 + 3) + (2 - 4) = \dots$$

In INFINITE sums however, this is not always true!

The error occurs in the third equality,

$$0 = 1 - 1 + 1 - 1 + 1 - 1 + \dots$$

The right hand side is an infinite series, and actually a geometric series with $a=1$ and $r=-1$

$$\sum_{n=1}^{\infty} (-1)^{n-1}$$

which we know is divergent. So we cannot say $0 = \sum_{n=1}^{\infty} (-1)^{n-1}$.