NAME: Solutions

MATH 172, Section 501; 10/30/2019

Quiz 8

Show your work! You may not use calculators, notes or books.

For each of the 4 series below, determine whether or not the series converges. Note that all series on this quiz have positive terms, so convergence for them is the same as absolute convergence. Justify your answer and any tests you use.

3 pts. 1. $\sum_{n=1}^{\infty} \frac{1}{n^4+5} = S$

Comparison Test: $\frac{1}{n^4+5} \leq \frac{1}{n^4}$, $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges (p-series w/ $p=4$) $\Rightarrow$ S is convergent by Comp. Test.

or Limit Comparison Test $a_n = \frac{1}{n^4+5}$, $b_n = \frac{1}{n^4}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^4}{n^4+5} = 1 \in (0,\infty)$$

$\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges $\Rightarrow$ S is convergent by LCT.

2 pts. 2. $\sum_{n=1}^{\infty} \frac{n^2}{2n^3+1} = S$

Limit Comparison Test $a_n = \frac{n^2}{2n^3+1}$, $b_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^3}{2n^3+1} = \frac{1}{2} \in (0,\infty)$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic) $\Rightarrow$ S is divergent by LCT.

OR: $2n^3+1 \leq 4n^3$, for all $n \geq 1$

$\Rightarrow \frac{n^2}{2n^3+1} \geq \frac{n^2}{4n^3} = \frac{1}{4n}$

$\sum \frac{1}{4n}$ diverges (harmonic) $\Rightarrow$ S is divergent by CT.

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3 pts. 3. $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1} = S$

$\lim_{n\to\infty} \frac{n^2}{2n^2+1} = \frac{1}{2} \neq 0 \Rightarrow$ S is <u>divergent</u> by T4D.

2 pts. 4. $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} = S$

<u>Ratio Test</u>: $a_n = \frac{n^2}{(n+1)!}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)^2}{(n+2)!} \quad \frac{(n+1)!}{n^2}$$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{n^2(n+2)} = 0 < 1$$

$\Rightarrow$ S is <u>convergent</u> by Ratio Test.

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③ pts.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2+3} = S$

$$\left.\begin{array}{l} \frac{1}{n^2+3} \leq \frac{1}{n^2} \\ \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ converges (p-series w/ } p=2) \end{array}\right\} \Rightarrow S \text{ converges by the Comparison Test.}$$

OR

$$\left.\begin{array}{l} a_n = \frac{1}{n^2+3} \; ; \; b_n = \frac{1}{n^2} \\ \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2}{n^2+3} = 1 \in (0,\infty) \\ \sum \frac{1}{n^2} \text{ convergent p-series} \end{array}\right\} \Rightarrow S \text{ converges by LCT.}$$

② pts.

2. $\sum_{n=1}^{\infty} \frac{n^3}{5n^4+2}$

$$\left.\begin{array}{l} a_n = \frac{n^3}{5n^4+2} \; ; \; b_n = \frac{1}{n} \\ \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^4}{5n^4+2} = \frac{1}{5} \in (0,\infty) \\ \sum \frac{1}{n} \text{ diverges (harmonic)} \end{array}\right\} \Rightarrow S \text{ diverges by LCT.}$$

OR

$$\left.\begin{array}{l} 5n^4+2 \leq 6n^4 \text{, for all } n \geq 2 \\ \Rightarrow \frac{n^3}{5n^4+2} \geq \frac{n^3}{6n^4} = \frac{1}{6n} \\ \sum \frac{1}{6n} \text{ diverges (harmonic)} \end{array}\right\} \Rightarrow S \text{ diverges by CT.}$$

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3 pts.

3. $\sum_{n=1}^{\infty} \frac{n^3}{5n^3+1} = S$

$$\lim_{n\to\infty} \frac{n^3}{5n^3+1} = \frac{1}{5} \neq 0 \Rightarrow S \text{ is divergent by T4D.}$$

2 pts.

4. $\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = S$

$$a_n = \frac{n}{(n+1)!}$$

$$\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{n+1}{(n+2)!} \frac{(n+1)!}{n}$$

$$= \lim_{n\to\infty} \frac{n+1}{n(n+2)} = 0 < 1$$

$\Rightarrow S$ is convergent by the Ratio Test.