NAME: Solutions

MATH 172, Section 501; 11/6/2019

Quiz 9

Show your work! You may not use calculators, notes or books.

(6) **1.** Find the radius of convergence and the interval of convergence for the power series:

$$\sum_{n=1}^{\infty} \frac{8^n x^n}{n^3}.$$

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{8^{n+1}x^{n+1}}{(n+1)^3}\cdot\frac{n^3}{8^n\cdot x^n}\right| = \lim_{n\to\infty}\frac{n^3}{(n+1)^3}\cdot 8\cdot|x|$$ (1 pt: correct simplif.)

(1 pt: Applying Ratio Test)

$$= 8|x| < 1 \iff |x| < \frac{1}{8} \Rightarrow R = \frac{1}{8}$$ (1 pt.)

(1 pt: correct limit)

Endpoints:

$x = \frac{-1}{8}$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ convergent b/c it is absolutely convergent;

$x = \frac{1}{8}$: $\sum_{n=1}^{\infty}\frac{1}{n^3}$ convergent p-series $(p=3)$

Endpoints: (1pt)

$\Rightarrow$ Interval $I = [-1/8, 1/8]$ (1 pt.)

(4) **2.** Use the geometric series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \ \forall |x| < 1,$$

to find a power series representation for the function

$$f(x) = \frac{1}{2+x^2}.$$

State for which values of x your formula is valid.

$$f(x) = \frac{1}{2+x^2} = \frac{1}{2}\cdot\frac{1}{1+\frac{x^2}{2}} = \frac{1}{2}\cdot\frac{1}{1-\left(-\frac{x^2}{2}\right)}$$ ← (2 pts. getting to this form)

$$= \frac{1}{2}\cdot\sum_{n=0}^{\infty}\left(-\frac{x^2}{2}\right)^n, \quad \forall \left|\frac{x^2}{2}\right| < 1 \text{ or } |x^2| < 2 \text{ or } |x| < \sqrt{2}$$

(1 pt.) (1 pt.)

$$\Rightarrow \boxed{f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n\cdot x^{2n}}{2^{n+1}}, \quad \forall |x| < \sqrt{2}}$$

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Quiz 9

Show your work! You may not use calculators, notes or books.

(6) **1.** Find the radius of convergence and the interval of convergence for the power series:

$$\sum_{n=1}^{\infty} \frac{n}{3^n}(x+2)^n.$$

Ratio Test: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{n+1}{3^{n+1}}(X+2)^{n+1}\cdot\frac{3^n}{n}\frac{1}{(X+2)^n}\right|$

(1pt: applying Ratio Test)

$= \lim_{n\to\infty}\frac{n+1}{n}\frac{1}{3}|X+2| = \frac{|X+2|}{3} < 1 \Rightarrow |X+2|<3$ $R=3$ (1 pt.)

(1pT.: correct simplif.) (1pt. correct limit)

$-3<X+2<3$

$-5<X<1$

Endpoints:

$X=-5$: $\sum_{n=1}^{\infty}\frac{n}{3^n}(-3)^n = \sum_{n=1}^{\infty}(-1)^n n$ divergent by T4D: $\lim_{n\to\infty}((-1)^n n)$ DNE

$X=1$: $\sum_{n=1}^{\infty}\frac{n}{3^n}3^n = \sum_{n=1}^{\infty} n$ divergent by T4D: $\lim_{n\to\infty}(n)=\infty$.

Endpoints (1 pt.)

$\Rightarrow$ Interval: $I=(-5,1)$ (1 pt.)

(4) **2.** Use the geometric series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ \forall |x|<1,$$

to find a power series representation for the function

$$f(x) = \frac{1}{2+x^2}.$$

State for which values of x your formula is valid.

Same as S501.