Name: ______________________________

May 6^{th}, 2016.
Math 2551; Sections L1, L2, L3.
Georgia Institute of Technology
Final Exam

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged: ______________________________

Problem	Possible Score	Earned Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
Total	140	

Remember that you must SHOW YOUR WORK to receive credit!

Good luck!

1. Find the minimum and maximum value of $f(x, y) = x^2 + y^2$ subject to the constraint

$$x^2 - 8x + y^2 - 4y = 0.$$

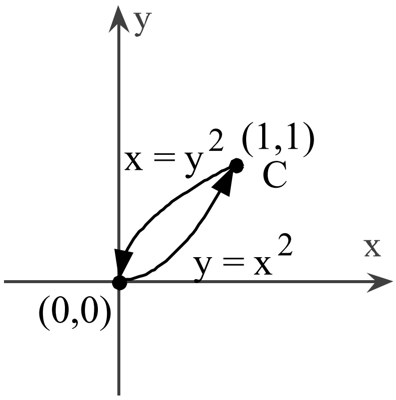
Indicate the points where these extreme values occur.

2. (a). Find the work done by the field $\mathbf{F}(x, y) = \langle 2x - 5y, 5x - 2y \rangle$ in moving a particle once counterclockwise around the curve $(x-1)^2 + (y-1)^2 = 1$.

(b). Find the outward flux of the field

$$\mathbf{F}(x, y) = \left\langle 4xy + y^2,\ 4x - y \right\rangle,$$

across the curve C pictured below.



3. Find all the critical points of the function:

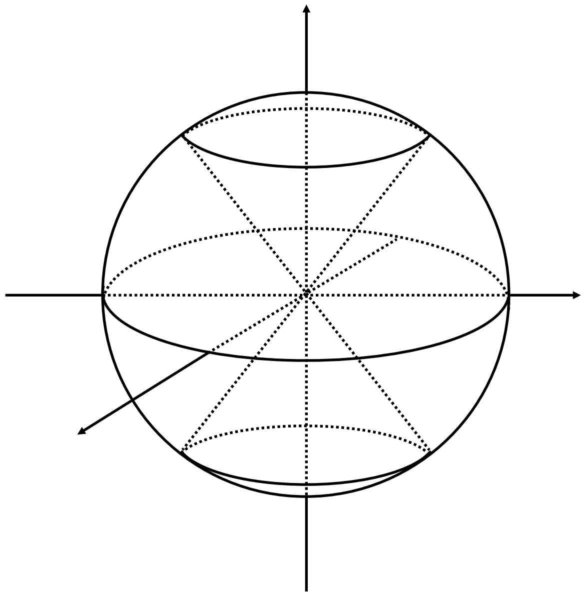
$$f(x, y) = x^3 - 3xy + y^3,$$

and classify each one as a local minimum, a local maximum, or a saddle point.

4. (a). Let x be a real number. What is $\sqrt{x^2}$?

$$\sqrt{x^2} =$$

(b). Let D be the solid that lies *inside* the sphere $x^2 + y^2 + z^2 = 2$ and *outside* the cone $z^2 = x^2 + y^2$. Use *spherical* coordinates to set up the triple integral that gives the volume of D. You do not need to compute the volume.



5. (a). Sketch the region of integration and convert the Cartesian integral:

$$\int_{-6}^{6} \int_{-\sqrt{36-x^2}}^{\sqrt{36-x^2}} \ln(x^2 + y^2 + 1)\, dy\, dx$$

into an equivalent polar integral. You do not need to compute the integrals.

(b). Sketch the region of integration and convert the polar integral:

$$\int_{0}^{\pi/4} \int_{0}^{7 \sec\theta} r^5 \cos^2\theta \, dr\, d\theta$$

into an equivalent Cartesian integral. You do not need to compute the integrals.

6. Consider the surface S given by the parametrization:

$$\mathbf{r}(\theta, z) = \left\langle 2\sin(2\theta),\ \ 4\sin^2\theta,\ \ z \right\rangle.$$

(a). Find a Cartesian equation for the surface S and sketch it. (Recall that $\sin(2\theta) = 2\sin\theta\ \cos\theta$).

(b). Find the equation of the plane tangent to the surface S at the point $P_0(\sqrt{3}, 1, 1)$ corresponding to $(\theta, z) = (\pi/6, 1)$.

7. Find the following limits:

a). $\lim_{(x,y)\to(0,0)} \frac{4e^y \sin(4x)}{5x}$

b). $\lim_{(x,y)\to(3,1)} \frac{xy - 3y - 5x + 15}{x - 3}$.

c). Show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{y^4 - 2x^2}{y^4 + x^2}.$$

8. Consider the curve:

$$\mathbf{r}(t) = \langle 4\cos t,\ \ 4\sin t,\ \ 3t\rangle\,,$$

and its unit tangent and unit normal vectors:

$$\mathbf{T} = \left\langle -\frac{4}{5}\sin t,\ \ \frac{4}{5}\cos t,\ \ \frac{3}{5}\right\rangle,$$

$$\mathbf{N} = \langle -\cos t,\ \ -\sin t,\ \ 0\rangle\,.$$

a). Find the unit binormal vector **B**.

b). Find the torsion τ along this curve.

9. (a). Write parametric equations for the line joining the points $(0, 2, 0)$ and $(2, 0, 0)$.

(b). Let C be the line segment from $(0, 2, 0)$ to $(2, 0, 0)$. Compute the line integral:

$$\int_C (x + y)\, ds.$$

10. Consider the lines:

$$\text{L1:} \quad x = -1 + 3t, \quad y = 2 + 3t, \quad z = 1 - t;$$

$$\text{L2:} \quad x = 1 - 4s, \quad y = 1 + 2s, \quad z = 2 - 2s.$$

(a). Find the point of intersection of these lines.

(b). Find an equation of the plane determined by the two lines.

11. Consider the curve:

$$\mathbf{r}(t) = \langle -\sin t,\ \cos t,\ 2t \rangle .$$

(a). Find the arc length parameter along this curve, taking $(0,1,0)$ for the initial point.

(b). Find the length of the portion of this curve with $0 \le t \le \pi/6$.

(c). Find the point on this curve that is at distance $\sqrt{5}\pi/2$ units along the curve from $(0,1,0)$ in the direction of increasing arc length.

12. Given that for a curve $\mathbf{r}(t)$:

$$\frac{d\mathbf{r}}{dt} = t(e^{t^2} + 1)\,\mathbf{i} + \frac{e^t}{e^t + 1}\,\mathbf{j} + 0\,\mathbf{k},$$

and that:

$$\mathbf{r}(0) = \langle 1, 0, 2 \rangle\,,$$

find $\mathbf{r}(t)$.

13. Find the outward flux of the field:

$$\mathbf{F}(x, y, z) = \langle x^4,\ -4x^3y,\ 3xz \rangle,$$

across the boundary of the region D cut from the first octant (where x, y, z are all positive) by the sphere $x^2 + y^2 + z^2 = 4$.

14. Prove that

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}.$$