Name: Solutions

January 28th, 2015.
Math 2401; Sections K1, K2, K3.
Georgia Institute of Technology
Exam 1

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged: ______________________

Problem	Possible Score	Earned Score
1	20	
2	18	
3	14	
4	17	
5	10	
6	21	
Total	100	

Remember that you must SHOW YOUR WORK to receive credit!

Good luck!

1. [20 pts.] Consider the points in space:

$$P(1,2,0);\quad Q(3,1,2);\quad R(-2,0,1).$$

a). [6 pts.] Express the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ in standard component form.

$\overrightarrow{PQ} = \langle 2, -1, 2 \rangle$ (3pts.) – 1pt. each

$\overrightarrow{PR} = \langle -3, -2, 1 \rangle$ (3pts.) – 1pt. each

b). [7 pts.] Find:

$$\overrightarrow{PQ} \times \overrightarrow{PR}.$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 2 \\ -3 & -2 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 2 \\ -3 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -1 \\ -3 & -2 \end{vmatrix}$$

(1pt.) – setting up determinant

$= \langle 3, -8, -7 \rangle$ (6pts.) 2pts. each

c). [7 pts.] Find an equation for the plane determined by the points P, Q and R. You do not need to simplify.

Normal Vector: $\overrightarrow{PQ} \times \overrightarrow{PR} = \langle 3, -8, -7 \rangle$ (1pt.)

Point P: $3(x-1) - 8(y-2) - 7(z) = 0$

Point Q: $3(x-3) - 8(y-1) - 7(z-2) = 0$

Point R: $3(x+2) - 8(y) - 7(z-1) = 0$

either one (6pts.)

2. [18 pts.] Find parametric equations for the line that is tangent to the curve:

$$\vec{r}(t) = (2\sin(t))\,\vec{i} + (t^4 - 5\cos(t))\,\vec{j} + (4e^{2t})\,\vec{k},$$

at the point on the curve where $t = 0$.

$\vec{v}(t) = \langle 2\cos t,\ 4t^3 + 5\sin t,\ 8e^{2t} \rangle$ (6pts.) - 2pts. each

$\vec{v}(0) = \langle 2, 0, 8 \rangle$ vector parallel to the line (3pts.)

$\vec{r}(0) = \langle 0, -5, 4 \rangle$ point on the line : $(0, -5, 4)$ (3pts.)

Parametric Equations:
(6pts.)
2 pts. each

$$\begin{cases} x = 2t \\ y = -5 \\ z = 4 + 8t \end{cases}$$

3. [14 pts.] Consider the vectors:

$$\vec{u} = \langle 1, 1, 1 \rangle\,,$$
$$\vec{v} = \langle 2, 1, 0 \rangle\,.$$

a). [7 pts.] Find the dot product $\vec{u} \cdot \vec{v}$.

$\vec{u} \cdot \vec{v} = 2 + 1 = \boxed{3}$ (7pts.)

b). [7 pts.] Find the angle θ between $\vec{u}$ and $\vec{v}$. Give an exact answer.

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$ (1pt.) formula

$|\vec{u}| = \sqrt{3}$ (2pts.)

$|\vec{v}| = \sqrt{5}$ (2pts.)

$\cos\theta = \dfrac{3}{\sqrt{3}\sqrt{5}} = \dfrac{\sqrt{3}}{\sqrt{5}}$ (1pt.)

$\boxed{\theta = \cos^{-1}\left(\dfrac{\sqrt{3}}{\sqrt{5}}\right)}$ (1pt.)

4. [17 pts.] Given that:

$$\frac{d\vec{r}}{dt} = (6\sqrt{t+1})\,\vec{\imath} + (e^{-t})\,\vec{\jmath} + \left(\frac{1}{t+1}\right)\vec{k},$$

$$\vec{r}(0) = \vec{k},$$

find $\vec{r}(t)$.

$$\vec{r}(t) = \int \frac{d\vec{r}}{dt}$$

$$= \left\langle \int 6\sqrt{t+1}\,dt,\ \int e^{-t}dt,\ \int \frac{1}{t+1}dt \right\rangle$$

$$= \left\langle 6\cdot\frac{2}{3}(t+1)^{3/2} + C_1,\ -e^{-t} + C_2,\ \ln(t+1) + C_3 \right\rangle \quad (7\text{pts.})$$

(2pts.) for each component

(1pt.) for adding constants

$$= \left\langle 4(t+1)^{3/2} + C_1,\ -e^{-t} + C_2,\ \ln(t+1) + C_3 \right\rangle$$

$$\Rightarrow \left.\begin{aligned} \vec{r}(0) &= \langle 4 + C_1,\ -1 + C_2,\ C_3 \rangle \\ &= \langle 0,\ 0,\ 1 \rangle \end{aligned}\right\} \Rightarrow \left.\begin{aligned} C_1 &= -4 \\ C_2 &= 1 \\ C_3 &= 1 \end{aligned}\right\} (3\text{pts.})$$

−(1pt.) for each constant

$$\boxed{\vec{r}(t) = \left\langle 4(t+1)^{3/2} - 4,\ -e^{-t} + 1,\ \ln(t+1) + 1 \right\rangle}$$

5. [10 pts.] Find the length of the curve:

$$\vec{r}(t) = (t\cos(t))\,\vec{i} + (t\sin(t))\,\vec{j} + \left(\frac{2\sqrt{2}}{3}t^{3/2}\right)\vec{k},$$

from the point $(0,0,0)$ to the point $(-\pi, 0, \frac{2\sqrt{2}}{3}\pi^{3/2})$.

$\vec{r}(0) = \langle 0,0,0\rangle \Rightarrow t=0$ start point (1pt.)

$\vec{r}(\pi) = \langle -\pi, 0, \frac{2\sqrt{2}}{3}\pi^{3/2}\rangle \Rightarrow t=\pi$ end point (1pt.)

$\vec{v}(t) = \left\langle \cos t - t\sin t,\ \sin t + t\cos t,\ \frac{2\sqrt{2}}{3}\cdot\frac{3}{2}t^{1/2}\right\rangle$ (3pts.) – 1pt. each

$= \langle \cos t - t\sin t,\ \sin t + t\cos t,\ \sqrt{2}\sqrt{t}\rangle$

$$|\vec{v}(t)|^2 = \cos^2 t - \cancel{2t\sin t\cos t} + t^2\sin^2 t + \sin^2 t + \cancel{2t\sin t\cos t} + t^2\cos^2 t + 2t$$ (1pt.) – simplification

$$= (\cos^2 t + \sin^2 t) + t^2(\sin^2 t + \cos^2 t) + 2t$$

$$= 1 + 2t + t^2$$

$$= (1+t)^2$$

(1pt.)

$|\vec{v}(t)| = |1+t| = \underline{1+t}$ for $0 \le t \le \pi$ (1pt.)

$$L = \int_0^{\pi} |\vec{v}(t)|\,dt = \int_0^{\pi} (1+t)\,dt = \left(t + \frac{t^2}{2}\right)\Big|_0^{\pi} = \boxed{\pi + \frac{\pi^2}{2}}$$ (1pt.) final answer

(1pt.) – formula

6. [21 pts.] Given the curve:

$$\vec{r}(t) = \langle t,\ 2\sin(t),\ 2\cos(t)\rangle\,,$$

find:

a). [8 pts.] The unit tangent vector $\vec{T}(t)$.

$\vec{v}(t) = \langle 1,\ 2\cos t,\ -2\sin t\rangle$ (3 pts.)

$|\vec{v}(t)| = \sqrt{1 + 4\cos^2 t + 4\sin^2 t} = \sqrt{1 + 4(\sin^2 t + \cos^2 t)} = \sqrt{5}$ (2 pts.)

$\vec{T} = \left\langle \frac{1}{\sqrt{5}},\ \frac{2\cos t}{\sqrt{5}},\ -\frac{2\sin t}{\sqrt{5}} \right\rangle$ (3 pts.)

$= \frac{1}{\sqrt{5}}\langle 1,\ 2\cos t,\ -2\sin t\rangle$

b). [8 pts.] The unit normal vector $\vec{N}(t)$.

$\frac{d\vec{T}}{dt} = \frac{1}{\sqrt{5}}\langle 0,\ -2\sin t,\ -2\cos t\rangle$ (3 pts.)

$\left|\frac{d\vec{T}}{dt}\right| = \frac{1}{\sqrt{5}}\sqrt{4\sin^2 t + 4\cos^2 t} = \frac{2}{\sqrt{5}}$ (2 pts.)

$\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|} = \langle 0,\ -\sin t,\ -\cos t\rangle$ (3 pts.)

c). [5 pts.] The unit binormal vector $\vec{B}(t)$. (You can use the back of this page for $\vec{B}$).

$\vec{B} = \vec{T} \times \vec{N}$ (1pt.) formula

(1pt.) - setting up determinant

$$= \frac{1}{\sqrt{5}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2\cos t & -2\sin t \\ 0 & -\sin t & -\cos t \end{vmatrix}$$

$$= \frac{1}{\sqrt{5}} \left\langle -2\cos^2 t - 2\sin^2 t,\ \cos t,\ -\sin t \right\rangle$$ (3pts.) - 1pt. each

$$= \frac{1}{\sqrt{5}} \left\langle -2,\ \cos t,\ -\sin t \right\rangle$$

Name: Solutions

February 18th, 2015.
Math 2401; Sections K1, K2, K3.
Georgia Institute of Technology
Exam 2

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Pledged:

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5	18	
6	15	
Total	100	

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Good luck!

[18 pts.]

1. (a). Find the limit, or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{3x^2}{x^2+2y^2}.$$

Linear paths: $y = kx$ (5 pts.)

$f(x,y) = \frac{3x^2}{x^2+2y^2}$; $f(x,y)\big|_{y=kx} = \frac{3x^2}{x^2+2k^2x^2} = \frac{3}{1+2k^2} \xrightarrow[x\to 0]{} \frac{3}{1+2k^2}$

$\Rightarrow$ Limit DNE by the Two-Path Test. (4 pts.)

(b). Find the limit, or show that it does not exist:

$$\lim_{(x,y)\to(4,1)} \frac{\sqrt{x}-2\sqrt{y}}{x-4y}.$$

$$\frac{\sqrt{x}-2\sqrt{y}}{x-4y} = \frac{\cancel{\sqrt{x}-2\sqrt{y}}}{\cancel{(\sqrt{x}-2\sqrt{y})}(\sqrt{x}+2\sqrt{y})} = \frac{1}{\sqrt{x}+2\sqrt{y}}$$ (5 pts.)

$$\lim_{(x,y)\to(4,1)} \frac{1}{\sqrt{x}+2\sqrt{y}} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$ (4 pts.)