Quiz 3

1. Consider the curve:

$$\mathbf{r}(t) = \langle t\sin t + \cos t,\ -t\cos t + \sin t\rangle;\ \sqrt{2} \le t \le 2.$$

a). (10 pts.) Find the unit tangent vector $\mathbf{T}(t)$.

b). (5 pts.) Find the length of the curve.

c). (5 pts.) Find the unit normal vector $\mathbf{N}(t)$.

(a). $\vec{v}(t) = \langle \sin t + t\cos t - \sin t,\ -\cos t + t\sin t + \cos t\rangle$

$\qquad = \langle t\cos t,\ t\sin t\rangle.$ **(5 pts.)**

$|\vec{v}(t)| = \sqrt{t^2\cos^2 t + t^2\sin^2 t} = \sqrt{t^2} = |t| = t \quad$ for $t \in [\sqrt{2}, 2]$ **(3 pts.)**

$\Rightarrow \boxed{\vec{T} = \langle \cos t,\ \sin t\rangle}$ **(2 pts.)**

(b). $L = \int_{\sqrt{2}}^{2} |\vec{v}(t)|\,dt = \int_{\sqrt{2}}^{2} t\,dt = \frac{1}{2}t^2 \Big|_{\sqrt{2}}^{2} = 2 - 1 = \boxed{1}$ **(5 pts.)**

(c). $\dfrac{d\vec{T}}{dt} = \langle -\sin t,\ \cos t\rangle \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} \Rightarrow \boxed{\vec{N} = \langle -\sin t,\ \cos t\rangle}$ **(5 pts.)**

$\left|\dfrac{d\vec{T}}{dt}\right| = 1$