

Quiz 4

(15 pts.)

1. For the functions below, find the partial derivatives with respect to each of the independent variables:

a). $w = ze^{2x-5y}$.

$$\frac{\partial w}{\partial x} = 2ze^{2x-5y}$$

3 pts.

$$\frac{\partial w}{\partial y} = -5ze^{2x-5y}$$

3 pts.

$$\frac{\partial w}{\partial z} = e^{2x-5y}$$

3 pts.

b). $z = x \sin(x^2 + 3y)$.

$$\frac{\partial z}{\partial x} = \sin(x^2 + 3y) + 2x^2 \cos(x^2 + 3y)$$

3 pts.

$$\frac{\partial z}{\partial y} = 3x \cos(x^2 + 3y)$$

3 pts.

(5 pts.)

2. If

$$g(s, t) = f(e^{t-s}, e^{s-t}),$$

where f is a differentiable function, find:

$$\frac{\partial g}{\partial s} + \frac{\partial g}{\partial t}$$

Hint: Don't panic because you don't know exactly what the partial derivatives of f look like - it works out.

$$g(s, t) = f(x, y), \text{ where } x(s, t) = e^{t-s}; y(s, t) = e^{s-t}$$

1/2 pt.

Chain Rule: $\frac{\partial g}{\partial s} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s} = -e^{t-s} \frac{\partial f}{\partial x} + e^{s-t} \frac{\partial f}{\partial y}$

2 pts.

$$\frac{\partial g}{\partial t} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t} = e^{t-s} \frac{\partial f}{\partial x} - e^{s-t} \frac{\partial f}{\partial y}$$

2 pts.

$$\oplus \quad \frac{\partial g}{\partial s} + \frac{\partial g}{\partial t} = \boxed{0}$$

1/2 pt.