$f(x,y) = 2x^2 + 2y^2 - x + y$

$\Omega = \{(x,y) : x^2 + y^2 \le 1\}$

Critical Points: $f_x = 4x - 1$, $f_y = 4y + 1$ $\quad \boxed{(1/4, -1/4)}$ - in the interior of Ω

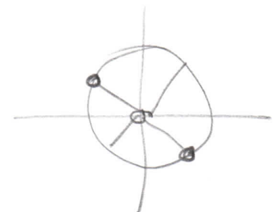
$$f(1/4, -1/4) = 2\cdot\frac{1}{16} + 2\cdot\frac{1}{16} - \frac{1}{4} - \frac{1}{4} = -1/4$$

Boundary: Solution 1 (Polar Coordinates)

$x = \cos\theta$, $y = \sin\theta$ $\Rightarrow$ $f(\theta) = 2\cos^2\theta + 2\sin^2\theta - \cos\theta + \sin\theta$

$= 2 - \cos\theta + \sin\theta \qquad \theta \in [0, 2\pi]$,

$$f'(\theta) = \sin\theta + \cos\theta$$



Solve $\sin\theta + \cos\theta = 0$ on $[0, 2\pi]$

$\tan\theta + 1 = 0$

$\tan\theta = -1 \qquad \boxed{\theta \in \{3\pi/4, 7\pi/4\}}$

Find interior critical point & evaluate f there - 8 pts

Find local min/max on the boundary - 10 pts

Final answer - 2 pts

$f(3\pi/4) = 2 - (-1/\sqrt{2}) + (1/\sqrt{2}) = 2 + \sqrt{2}$

$f(7\pi/4) = 2 - 1/\sqrt{2} + (-1/\sqrt{2}) = 2 - \sqrt{2}$

$f(0) = f(2\pi) = 2 - 1 = 1$

$\Rightarrow$ Absolute max: $\boxed{2+\sqrt{2}}$; Absolute min: $\boxed{-1/4}$

Solution 2 (Lagrange multipliers)

$\nabla f = \langle 4x - 1, 4y + 1\rangle$

$\nabla g = \langle 2x, 2y\rangle$

$$\begin{cases} 4x - 1 = 2\lambda x \\ 4y + 1 = 2\lambda y \\ x^2 + y^2 = 1 \end{cases} \qquad \left.\begin{aligned} (4-2\lambda)x = 1 &\Rightarrow x = \frac{1}{4-2\lambda} \\ (4-2\lambda)y = -1 &\Rightarrow y = \frac{-1}{4-2\lambda} \end{aligned}\right\} \Rightarrow y = -x$$

$2x^2 = 1 \Rightarrow x = \pm 1/\sqrt{2} \Rightarrow (1/\sqrt{2}, -1/\sqrt{2})$ or $(-1/\sqrt{2}, 1/\sqrt{2})$

$f(1/\sqrt{2}, -1/\sqrt{2}) = 2\cdot\frac{1}{2} + 2\cdot\frac{1}{2} - \sqrt{2} = \boxed{2-\sqrt{2}}$; $f(-1/\sqrt{2}, 1/\sqrt{2}) = \boxed{2+\sqrt{2}}$