NAME: Solutions

Math 2551 (L1-L3)
3/16/2016

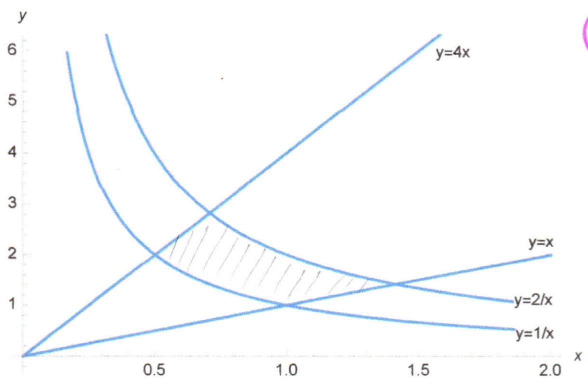
Quiz 7

1. Compute

$$\iint_R y^2\,dA,$$

where R is the region in Quadrant I of the (x, y)-plane bounded by the curves $y = 1/x$, $y = 2/x$, $y = x$, and $y = 4x$ (see figure). You can use the substitution:

$$u = \sqrt{xy};\quad v = \sqrt{\frac{y}{x}}.$$



5pts. Solve for x and y:

$xy = u^2$

$y/x = v^2 \Rightarrow y = xv^2 \Rightarrow x^2v^2 = u^2 \Rightarrow x = u/v$

$\Rightarrow y = uv$

$$\begin{cases} x = \dfrac{u}{v} \\ y = uv \end{cases}$$

5pts. $\Rightarrow$ Jacobian: $J = \begin{vmatrix} 1/v & -\dfrac{u}{v^2} \\ v & u \end{vmatrix} = \dfrac{u}{v} + \dfrac{u}{v} = \dfrac{2u}{v}$

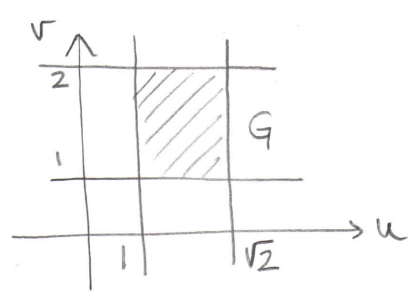
Transform the region: 5pts.

$y = \frac{1}{x} \Rightarrow xy = 1 \Rightarrow u^2 = 1 \Rightarrow u = 1$

$y = \frac{2}{x} \Rightarrow xy = 2 \Rightarrow u^2 = 2 \Rightarrow u = \sqrt{2}$

$y = x \Rightarrow \frac{y}{x} = 1 \Rightarrow v^2 = 1 \Rightarrow v = 1$

$y = 4x \Rightarrow \frac{y}{x} = 4 \Rightarrow v^2 = 4 \Rightarrow v = 2$



Transform the integral: 5pts.

$$\iint_R y^2\,dA = \iint_G (uv)^2\,\frac{2u}{v}\,dA$$

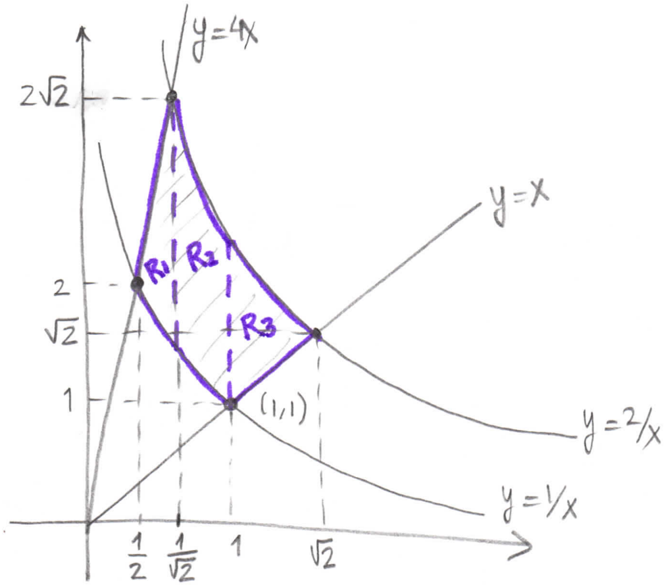
$$= 2\int_1^2 \int_1^{\sqrt{2}} u^3 v\,du\,dv$$

$$= 2\left(\frac{u^4}{4}\Big|_1^{\sqrt{2}}\right)\left(\frac{v^2}{2}\Big|_1^{2}\right)$$

$$= 2\left(1 - \frac{1}{4}\right)\left(2 - \frac{1}{2}\right)$$

$$= 2 \cdot \frac{3}{4} \cdot \frac{3}{2}$$

$$= 9/4$$



<u>Solution II</u> : If you want to use Cartesian coordinates ...

$(y=\frac{1}{x}) \cap (y=x)$: $x=1$; $y=1$ $\qquad$ $(y=\frac{1}{x}) \cap (y=4x)$; $x=\frac{1}{2}$; $y=2$

$(y=\frac{2}{x}) \cap (y=x)$: $x=\sqrt{2}$; $y=\sqrt{2}$ $\qquad$ $(y=\frac{2}{x}) \cap (y=4x)$; $x=\frac{1}{\sqrt{2}}$; $y=2\sqrt{2}$

Integrate over R in Cartesian coordinates: $\iint_R y^2 dA = \left(\iint_{R_1} + \iint_{R_2} + \iint_{R_3}\right) y^2 dA$

- $$\iint_{R_1} y^2 dA = \int_{1/2}^{1/\sqrt{2}}\int_{1/x}^{4x} y^2\,dy\,dx = \int_{1/2}^{1/\sqrt{2}} \frac{y^3}{3}\Big|_{1/x}^{4x} dx = \int_{1/2}^{1/\sqrt{2}} \left(\frac{64x^3}{3} - \frac{1}{3x^3}\right)dx$$
$$= \frac{1}{3}\left(16x^4 + \frac{1}{2x^2}\right)\Big|_{1/2}^{1/\sqrt{2}} = \frac{1}{3}\left(4 + \cancel{1} - \cancel{1} - 2\right) = \frac{2}{3}$$

- $$\iint_{R_2} y^2 dA = \int_{1/\sqrt{2}}^{1}\int_{1/x}^{2/x} y^2\,dy\,dx = \int_{1/\sqrt{2}}^{1} \frac{1}{3}y^3\Big|_{1/x}^{2/x} dx = \frac{1}{3}\int_{1/\sqrt{2}}^{1} \frac{8}{x^3} - \frac{1}{x^3}\,dx$$
$$= \frac{1}{3}\int_{1/\sqrt{2}}^{1} \frac{7}{x^3}dx = \frac{7}{3}\cdot\frac{-1}{2x^2}\Big|_{1/\sqrt{2}}^{1} = \frac{7}{3}\left(-\frac{1}{2}+1\right) = \frac{7}{6}$$

- $$\iint_{R_3} y^2 dA = \int_{1}^{\sqrt{2}}\int_{x}^{2/x} y^2\,dy\,dx = \int_{1}^{\sqrt{2}} y^3/3\Big|_{x}^{2/x} dx = \frac{1}{3}\int_{1}^{\sqrt{2}}\left(\frac{8}{x^3} - x^3\right)dx$$
$$= \frac{1}{3}\left(-\frac{4}{x^2} - \frac{x^4}{4}\right)\Big|_{1}^{\sqrt{2}} = \frac{1}{3}\left(-2-1+4+\frac{1}{4}\right) = \frac{5}{12}$$

$$\Rightarrow \iint_R y^2 dA = \frac{2}{3} + \frac{7}{6} + \frac{5}{12} = \frac{8+14+5}{12} = \frac{27}{12} = \frac{9}{4}$$